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MATHEMATICS IN THE ELEMENTARY SCHOOLS OF THE UNITED STATES

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

THE AMERICAN REPORT

COMMITTEES I AND II

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THE AMERICAN REPORT

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The following report of Subcommittee I was prepared to meet the special needs of foreign readers. Although it does not relate solely to elementary schools, it forms an appropriate introduction to the general report of Committee No. I, since some schematic survey of our educational institutions is necessary in beginning the study of our system. For this reason it precedes the general report of this committee.

GENERAL DIVISIONS.

The American system of education divides into two main divisions:

A. Public education: Government controlled and Government supported.

B. Nonpublic education: Controlled by religious bodies, private corporations, or persons with philanthropic or fiduciary intent. May be subdivided into: (*a*) Religious education, controlled and supported by religious denominations and sects; (*b*) private and semiprivate institutions, including those managed for profit; and (*c*) supplementary and extra institutionalized agencies, such as the Carnegie foundation for the promotion of teaching, the Russell Sage foundation, and the National Education Association.

AGENCIES OF PUBLIC EDUCATION.

I. The National Government. This administers but a small portion of public education. (*a*) The Bureau of Education is a national office, under the Department of the Interior. It is designed mainly to assemble, digest, and disseminate educational information. It is also charged with certain responsibilities for native education (in Alaska). (*b*) The Indians of the United States are mostly wards of the Nation. As such they are educated by the National Government, either through subsidies granted to church bodies or through schools maintained by the Department of the Interior. These schools partake of an industrial character. (*c*) The National Government also organizes or provides for the organization of the school systems for its dependencies, such as Porto Rico and the Philippines. Similarly it provides for the school system of the Capital City—Washington. (*d*) Two principal schools—West Point and Annapolis—and a number of minor schools are maintained for the

training of officers and men for the Army and Navy. (c) The National Government makes appropriations to each of the States for the furtherance of higher education in agriculture and the mechanic arts. It has also in the past provided extensive land grants for the support of the public-school systems of the States and universities. In conjunction with the States, or sometimes philanthropic agencies, it carries on investigation and experimental work in agriculture for educational purposes.

II. The State governments. The primary agency for the exercise of educational functions is the State. The National Government exercises no control and so far little power of suggestion over State governments.

(a) State constitutions. These, as fundamental laws, usually contain provisions requiring the organization of systems of public education, and often defining special features.

(b) State legislatures. These, meeting usually annually or biennially, pass laws fixing, often in great detail, forms of educational administration, providing for support and the like.

(c) State board and State superintendent. Nearly all States have provisions for a State machinery of educational administration, the scope and authority of which vary greatly. In general, this administration is confined to elementary and secondary education. In many cases the State superintendent is elected by general suffrage, and his functions are mainly supervisory and advisory. In other cases he is an educational expert, appointed by a State board, and with considerable authority for imposing standards and administering types of education. State boards also vary widely in functions. Some have authority to select a variety of educational experts, to direct the expenditure of funds, and to control certain types of education. Others have functions limited to custody of State school funds and the certification of teachers.

(d) State educational institutions. The State forms the area for the administration of a variety of educational institutions. 1. Universities and colleges. Most Western and Southern States support and control universities; and all have an agricultural college, partly supported by national grants. Frequently these are directed by special boards appointed by the State executive. 2. Normal schools are found in all but two States, governed either by a State board or by local boards appointed for this purpose by the executive or by a combination of both authorities. 3. Schools for delinquents, defectives, and dependents. Almost everywhere most of these are State institutions; but sometimes the State supports and leaves control to a religious or philanthropic body. 4. Special schools, especially for agricultural, forestry, or technical training, maintained as State institutions, are found in a few States. 5. In some States a library commission is maintained as part of the State educational machinery.

(e) Teachers' institutes. State libraries for teachers and extension work are in some cases organized and supported by State authorities, but more commonly by smaller areas than the State.

(f) In a few States, State authorities certificate teachers, inspect schools (especially secondary schools), select textbooks, approve plans for buildings, examine pupils (especially for admission to secondary school or college), and provide or suggest courses of study. Such functions are found in relatively few States, but there is a growing tendency in this direction.

III. Local areas of administration of education. These vary in size and political importance in the various States. Five types are commonly found:

(a) The county, the largest subdivision of the State, under the law exercises little or no educational function in New England, much in most Southern States, and a varying amount in North Central and Western States. Is of considerable importance as an authority for inspection (commonly miscalled supervision), auditing of educational accounts, certification, after training of teachers, and of taxation. Less frequently it is the administrative unit for secondary agricultural education; occasionally for general secondary education.

(b) The town or township. Usually embraces several schools and is frequently the area of local administration and taxation in all that pertains directly to secondary and elementary schools.

(c) The district. Originally the area from which one school drew its pupils. In early stages of the evolution of educational administration it had important functions, apart from the direct management of schools. These functions are gradually being transferred to larger areas. They are the consolidation of districts, and the transportation of pupils. In many States large central schools are being formed.

(d) The city. Municipal areas give rise to districts with special characteristics. The city frequently combines the administrative functions exercised by county, town, and district, and in some cases the State even relaxes its supervision in the case of city districts. Within the cities of the United States educational administration tends to become centralized, lay boards becoming smaller, the authority of educational experts greater.

(e) In a few States special administrative ways exist. In Massachusetts a union of towns forms a supervisory district; in New York the school commissioner district differs from the county; in Virginia the State is divided into a few large areas for certain kinds of supervision.

TYPES OF PUBLIC SCHOOLS.

I. The kindergarten, usually admitting pupils from 3 to 5 years of age, and seldom holding them beyond 6. Found mainly in cities, and most commonly provided in poorer sections. In 1907–8 all cities having 8,000 or more population reported their kindergartens as containing—157,000 pupils. Nowhere is kindergarten training prerequisite to entrance to the elementary school.

II. The elementary school. This term is now commonly applied to the types of schools formerly designated as primary, intermediate, and grammar or grammar grades. The course is nearly always 8 years in length, very rarely 7, occasionally 9 or 10. The completion of the elementary course is necessary for entrance to the high schools, and the elementary school rarely provides work paralleling that of the secondary school, this providing the marked contrast to European practice. In a few cases it is attempted to teach secondary subjects, such as foreign language and mathematics, in the upper grades of the elementary school. Under present conditions the standard maintained by the elementary school is such that seldom more than half of all children are able to finish it by the close of the compulsory school period—14 years of age. In 1907–8 some 16,100,000 children were reported in the elementary schools.

III. Secondary schools under public support are commonly called high schools. Of these there are four or more distinguishable types: (*a*) General high schools, sometimes subdivided as classical, literary, scientific, etc., whose aim is primarily cultural, and whose curricula are mainly composed of languages, mathematics, science, and history. (*b*) Commercial high schools, sometimes within general high schools. These vary largely in character, some having short courses, other substantial four-year programs, whose controlling aim is preparation for commercial vocations. (*c*) Technical high schools, variously called manual training, mechanic arts, and polytechnic high schools, which aim to include with parts of a general course a considerable training in the technical subjects which attach to the industrial arts or household arts. (*d*) Agricultural high schools. These vary largely in curricula, some being ordinary high schools with attempts at agricultural training, others centering mainly around studies of agriculture.

In 1907–8 there were in the public high schools of the United States 790,000 students, or about 0.92 per cent of the total population; of these nearly 60 per cent were girls. Very few of these students were under 14 years of age, and also few were over 19. As contrasted with European secondary schools, the most significant fact is the late age at which pupils may take up secondary school work, it being obligatory to complete the elementary course first. About one-half the pupils entering high school do not stay more than two years, or beyond the age of 16. Of the above number of students, 59,000 were reported in commercial departments. No separate statistics are available of attendance in public technical secondary schools, but in 1907–8 there were in the public and private schools of this type 58,000 students.

IV. Normal schools, designed primarily for the training of teachers for elementary schools, are found in all but four States, where courses at State colleges are provided. In 1907–8 there were 189 of these State normal schools, with 64,000 students. Two types are distinguishable: Those receiving only high-school graduates, and usually having a course two years in length, and those receiving pupils with little more than elementary school preparation, and having courses from three to five (usually four) years in length. Sometimes both kinds of courses are found in the same school. The States better supplied with secondary schools now usually require high-school graduation for admission.

Many of the larger cities of the United States also maintain city training schools which receive high-school graduates and give them a one or two years' course in preparation for work in the elementary schools of the respective cities.

A number of normal schools, in addition to their two-year courses for high-school graduates, also maintain four-year courses for exceptional students, and give degrees. At least one (Albany Normal College, New York) aims primarily to prepare teachers for secondary schools. But a large and increasing majority of the teachers in secondary schools must be graduates of the regular colleges and universities.

Normal schools are frequently resorted to by young people whose primary aim is a liberal education, rather than a preparation for teaching.

V. Colleges and universities. Satisfactory statistics of public as distinguished from private colleges and universities do not exist. In 1907–8, of the 573 universities, colleges, and technological schools having 150,000 students, it was reported that 89 having over 60.000 students were public—that is, were supported mainly at public expense and were governed by public bodies; 62 of these were State universities or colleges of agriculture. The attendance at these institutions averages much higher than at private schools. Usually

their admission requirements coincide with the requirements for graduation from local secondary schools, thus forming part in a continuous chain of public education.

Affiliated with, or integrated with, many of the public universities are professional colleges for medicine, law, pharmacy, and the like. Within recent years there are also appearing special colleges as parts of the universities for the training of teachers, especially those for secondary schools, and those preparing for administrative work.

VI. Special schools. These exist to meet special class or social needs, and have their standards governed by the conditions to which they must adapt themselves.

(a) Under the National Government are two schools primarily to train officers for the Army and Navy. These are of collegiate or technological rank, and receive students of selected qualities. In addition, the Government maintains extension or graduate study for officers in service and a number of schools for the training of cadets or newly enlisted men for the Navy.

(b) Indian education is also under the direction of the National Government. Schools are maintained in Alaska, on the reservations in the United States, and in a number of cases off the reservations, for pupils especially of a more advanced type. The work in these schools is usually elementary in character, with increasing attention to practical or vocational aspects. In a few schools (Hampton, Haskell) special attention is given to the training of teachers, and especially for the transmission of the household arts and suitable vocations for men.

(c) Originally, in most of the States, philanthropy supported the education of some defectives, dependents, and delinquents. Then the States subsidized these institutions and later in many cases either took them over or supplanted them with public schools of day or institutional type. Some are under the State administrative machinery, others under local control and support, but these distinctions are unimportant. Each type of school works out courses of instruction and methods adapted to its needs. The work is increasingly vocational in character. In 1907–8 there were 40 State schools for the blind, with 4,340 pupils; 122 schools for the deaf (55 State, 51 public day schools, and 16 private), with 10,042 students in the State schools; and 25 State institutions of school character for the feeble-minded, with 17,225 inmates. In the same year 92 reform schools reported 36,900 pupils, nearly all from 12 to 16 years of age. The care of dependents is mostly under private auspices, and no satisfactory statistics are extant of the number of schools.

(d) Several States have established special forms of industrial schools, sometimes of secondary grade, sometimes intermediate between secondary school and college. There are no available statistics of these. As a rule their curricula approximate in character the technical secondary schools, with specially intensive developments along the lines of agriculture, industrial arts, and household arts.

(e) Continuation schools, evening schools. These are extensively developed in the larger cities of the United States. In 1905–6, 23 cities reported 314,000 children enrolled and 129,000 in average daily attendance. No satisfactory classification exists showing number who are taking technical work, high-school work, studies of an elementary nature, or special work in English for foreigners. All these

types are represented. It is generally known that special classes for the teaching of English to foreigners are largely on the increase and that technical classes are being steadily developed. But it is rare to find consistent programs running through a series of years—the work is fragmentary.

A form of educational work growing out of the universities is that called university extension. By means of lectures and extension classes it seeks to disseminate and popularize higher education.

NONPUBLIC AGENCIES OF EDUCATION.

I. Religious organizations. A very large part of private education in the United States from the lowest to the highest grade is carried on by religious organizations. With very few exceptions (in the case of schools for dependents, delinquents, and defectives) these schools receive no public aid; and those not receiving public aid are only rarely under public supervision, and that of the most perfunctory character. No system of certification prevails with regard to teachers in these schools; and they develop their own standards according to their own needs. Except for colleges and some forms of secondary education, it is not possible to procure statistics. In general it is true that the competition of the public schools tends to cause these private institutions to endeavor not to fall too far behind in the quality of their teaching. In these schools, especially those under the Roman Catholic Church, many of the teachers give their services as part of their religious duty, and where teachers work for pay, that is very low. As a rule, these schools are not experimental, but aim deliberately to parallel and if possible to fill the place of the public schools for certain classes of children.

II. Philanthropic organizations. These not infrequently grow out of church influences, but also quite commonly have no direct religious connection. On the whole they tend to develop types of education with which the system of public schools does not at present concern itself. These led the way in establishing higher education, in endeavoring to educate the defective, the delinquent, and the neglected; in establishing vocational education; and in developing the wider adaptations of education in vacation school, playground vocation bureaus, and so on. Consequently the schools under direction of nonreligious philanthropic agencies tend to be experimental, and their standards change, and the results of their work do not lend themselves to statistical treatment.

III. Commercial ventures. Some forms of education in the United States represent efforts for private profit. The types of school resulting vary largely in character and quality. Private secondary education usually caters to the wealthier classes of the community, especially in providing schools that combine residence and instruction. These schools have not State supervision, but tend, by virtue of their clientage, to be of excellent quality. Many of them have as their controlling function the fitting of students for college. Another type is that fitting for certain portions of commercial practice. These vary enormously, some being of excellent quality, others giving short courses and relying largely on advertising to win students. Of similar quality are a number of private normal schools, which aim to prepare students for teachers' examinations. Correspondence schools are private ventures, sometimes giving good work under adverse conditions, sometimes preying upon the credulity of young workingmen. A few private schools for industrial arts (trade schools) exist, but play a small part in vocational education.

The large majority of colleges and universities are private, and frequently originated under the auspices of some religious organization; but these are tending away from any form of religious control, and fall into the group described above as philanthropic, since only a small part of their actual expenses are usually derived from the fees of students.

Over private educational agencies of all sorts there is almost no State control, except in the case of those that deal with delinquent or afflicted children. True to the prevalent individualism of America, it is assumed by the State that the demands of those who patronize the private schools and the competition of those under public management is sufficient to insure the quality of the work of the former. In New York and Connecticut there are certain beginnings of State supervision, but they are not yet significant, except in the particular respect that scholarship standards in secondary education are made largely the same in New York State for both public and private schools by a system of examinations conducted by public authorities. But in such matters as certification of teachers, standards of attendance, equipment, and methods of teaching there exists no form of public control.

The exceptions are found in those institutions which care for destitute, defective, and delinquent children, and here State supervision is exercised primarily because these institutions receive considerable public aid. Those philanthropic institutions that do not obtain public assistance have little or no public inspection.

TYPES OF NONPUBLIC SCHOOLS.

These usually parallel the types of public school given above, except as regards special features, like the boarding life of the private secondary school, the short courses of the business school, and the religious education found in the schools under religious organizations.

I. Kindergartens. Almost always philanthropic and frequently contributing some training of teachers.

II. Parochial elementary schools. These are mainly developed under the Lutheran and Roman Catholic Churches, and usually parallel the public schools. In the Catholic schools the supervision is under the parish priest, and the teachers are often members of orders.

III. Private elementary schools maintained for profit are few and have small influence.

IV. Private business schools are conducted chiefly for profit. Over 150,000 students attended such schools in 1906–7. Admission requirements and quality of work vary enormously and generally tend to be strictly practical, i. e., produce in a short time the maximum of vocational efficiency.

V. Private trade schools. These are not numerous and vary greatly in character. A few are maintained by industries to assist apprentices, in which case the work is usually technical, to supplement shop work. In other cases schools maintained for profit give short and intensive training in certain trades (commonly woodworking, plumbing, machine-shop work, and so on). Several schools endowed by philanthropy give long courses of trade training.

VI. Correspondence schools. These probably have over a hundred thousand students in the United States (no exact figures available) to whom they teach mathematics, drawing, applied science, especially mechanics, and in less degree a few other subjects. Standards are variable; much of the work plays upon credulity, but under favorable conditions the more self-reliant students gain much. They exist mainly for profit.

VII. Parochial secondary education. This is yet little developed. Secondary education for religious bodies is accomplished mainly in academies and preparatory departments of denominational colleges.

VIII. Private secondary schools. These are numerous and strong. They fill two chief functions, sometimes not closely related. They serve as boarding schools at the adolescent period when many parents believe that boarding-school life is very valuable for their children; and they serve to prepare pupils directly for college. They grow but slowly, but have a fairly constant attendance of over 150,000 pupils. Many of them have endowed foundations.

IX. Colleges and universities. These seldom exist for profit, usually owing their foundation to religious or philanthropic zeal. In recent years religious control has greatly relaxed, so that it was easily possible for many of them to change some nominal restrictions in their organization and come under the definitions imposed by the Carnegie Foundation for the Advancement of Teaching.

In 1907–8 there were reported to the Commissioner of Education 484 private universities, colleges, and schools of technology, with enrollment of over 96,000 students. While a secondary education is assumed as prerequisite for admission, it is known that standards vary widely among these private institutions of higher learning.

Professional schools, often parts of universities but also in many cases independent, are, apart from those devoted to engineering pursuits, of six kinds—theological (9,500 students), law (18,000 students), medical (22,000 students), dental (6,500 students), pharmaceutical (5,500 students), and veterinary (2,200 students).

X. Special schools. A variety of special schools not included above exist under private leadership. The education of negroes, of immigrants, and of defectives has produced schools adapted to meet particular ends. Frequently these serve society as experimental stations, pointing the way to subsequent State action. Their character and purposes are too varied to admit of statistical treatment.

COMMITTEE NO. I. GENERAL ELEMENTARY SCHOOLS.

The special field of investigation assigned to this committee was the teaching of mathematics in both public and private schools of the rank known as elementary schools. It was desired that while the investigation should cover whatever ground seemed pertinent, it should especially consider the following topics:

(*a*) The aim and organization of the schools and the general relation of each type of school to the others; (*b*) the mathematical curriculum in other schools; (*c*) the question of examinations from the point of view of the schools; (*d*) the methods employed in teaching in the schools; (*e*) the preparation of teachers for the schools. It was desired that the report should particularly set forth the present conditions and tendencies. An attempt has been made to carry out the spirit of these directions. The report is organized with reference to these five points.

I. THE AIM AND ORGANIZATION OF THE ELEMENTARY SCHOOLS.

DEFINITION AND AIM OF ELEMENTARY EDUCATION.

Elementary education is that training which by common custom is deemed desirable and necessary for the duties and privileges of citizenship. It is usually made compulsory by law. Like all such definitions, the meaning of this statement becomes clear only through an examination of its interpretation as given in the curricula of the various school systems or in somewhat more specific statements of its content by various individuals. The elementary school strives to give the pupil control over the tools for the acquisition of knowledge, to develop interests and desires for further acquisition of knowledge, to excite a large number of curiosities, to stimulate thinking, to form the habit of collecting facts with which to think, and to aid the pupil in the formation of right habits of conduct. Prof. Thorndike has said:

> The special proximate aims of the first six years of school life are commonly taken to be to give physical training and protection against disease; knowledge of the simple facts of nature and human life; the ability to gain knowledge and pleasure through reading, and to express ideas and feelings through spoken and written language, music, and other arts; interest in the concrete life of the world; habits of intelligent curiosity; purpose in thinking, modesty, obedience, honesty, helpfulness, affection, courage, and justice, and the ideals proper to child life.

Prof. Hanus has said:

> Elementary, or presecondary school education, should provide the most salutary physical environment for the pupil, and promote his normal physical development through appropriate training. It should open the mind of the child and let the world in. It should stimulate and gratify curiosity in every field of worthy human activity, and utilize this curiosity for the acquisition of knowledge and the development of incipient permanent interests in and power over this knowledge. It should acquaint the pupil with his duties and his privileges as a temporarily dependent member of society, and promote the development of habits of thought and conduct in harmony with his growing insight.

ELEMENTARY SCHOOLS.

Different localities have interpreted this definition as seems proper to them, and in accord with the interpretation they have organized their elementary schools. In the Eastern States these schools often include the first nine school years or grades; in the South, usually seven years; in the North and West usually eight years. There is a movement urged by some of the best educators to have the elementary grades cover the first six school years, from the age of 6 to the age of 12, letting the secondary school cover the next six school years.

The school year consists of approximately 180 school days per year. In 1890 the average in cities of from 4,000 to 8,000 population was 180.9 days, and for the whole country 151 days. The school day is approximately five hours in length, and the school is in session from Monday to Friday, inclusive.

II. THE CURRICULUM IN MATHEMATICS IN THE ELEMENTARY SCHOOLS.

CURRICULUM IN GENERAL.

The term "arithmetic" is used to denote the mathematical element of the curriculum in American elementary schools. Arithmetic is taught in all of the schools. It may safely be said that the public at large consider it one of the most important subjects of the curriculum, and circumstances have given it in the past an unusually prominent place in the estimation of teachers. Mathematics was formerly, and is to-day, an excellent test of the general alertness and "brightness" of the pupils. As a result the "bright" pupils, who later become the teachers, continued to give emphasis to the subject. Furthermore, the organization of the rural school with its one teacher in charge of many classes necessitated much seat work, and since assignments in mathematics could readily be made and could be corrected very easily, while to have "ciphered through the book" was a mark of distinction, and since pupils who had any ambition to attain this distinction could thus be kept "in order" for long periods, mathematics came to form a major part of the curriculum in the rural school, and it has maintained its position to an extent up to the

present. There is a disposition in school circles to deny that the importance of mathematical training is such as to warrant the place it has occupied and still holds.

A study of the time devoted to various subjects in the grades in 50 of the leading cities showed that about 15.26 per cent of all of the school time was devoted to arithmetic. The same study indicated certain other facts bearing upon the place of mathematics in the curriculum. It showed that arithmetic as a formal study is being dropped out of the first and second grades and that in the upper grades arithmetic is being supplemented by algebra and geometry. Out of these 50 large school systems 19 only have algebra as such in the seventh or eighth grades and only 3 give any time or attention to geometry other than the work in mensuration. Recommendations have appeared from time to time during the past 15 years that the arithmetic course be curtailed and enriched—curtailed by omitting the study in the early grades and by omitting certain topics; enriched by introducing geometry, algebra, and other interesting topics in the upper grades. The statistics quoted above indicate that only 38 per cent of the 50 largest cities have followed out the recommendation. This same condition was found by another entirely independent investigation, except that even a smaller proportion of the schools were giving attention to algebra. So, in general, mathematics in American schools means particularly arithmetic with a certain amount of mensuration.

CONDUCT OF THE MATHEMATICS COURSE.

The teachers follow rather closely the textbook which has been provided for the schools. Whether following the text or independent of it, they are guided as to the extent of each year's work by an outline or course of study. The course of study is usually made out by the superintendent of schools of the city; in larger cities the superintendent delegates this work either to assistants or to a committee of principals or of principals and teachers, while occasionally the head of the department of mathematics in the high school is called upon to assist. The supervising officers of the schools are as a rule expert, experienced men and women, who have "risen to the top" because of their ability, and they bring to their work on the courses of study the product of their years of experience. While this is true, it is natural to inquire whether it is common to have in large systems an expert supervisor of mathematics, analogous to the supervisors of music and art. An investigation has shown that it is exceedingly rare for cities to employ such a supervisor of mathematics, although in some cases the work is delegated to an assistant superintendent. The same condition obtains in the case of geography, history, and other subjects. At the same time mathematics undoubtedly is a sub-

ject which calls for expert knowledge which is less commonly part of the equipment of all teachers; and, furthermore, it is a subject in the teaching of which there is to-day an unusual amount of change and uncertainty. It must be said that there are in all systems of schools a growing number of teachers who have made a special study of the teaching of mathematics in schools of education. The committee feels that expert supervision of arithmetic in the public schools should be more common.

COURSE OF STUDY.

The course of study in its simplest form consists merely of directions as to the page limits within the text, thus marking the assignment for each year. It varies from this to a much more detailed outline in communities where there is dissatisfaction with the general type of text. In these latter places there is given an outline by grades of the topics to be covered, together with teaching helps, references, and suggestions for problem material. In such cases it is generally understood that the outline rather than the text is to be followed. The wisdom of this plan depends upon the amount and character of the supervision, and upon the ability of the teaching force.

CONTENT OF THE CURRICULUM IN MATHEMATICS.

The following summary indicates the general content of the courses by grades:

Grade 1.—More or less incidental number work or number work correlated with manual training or with some other definite subject. Variations: From no number work at all to very formal work on addition, subtraction, and the multiplication tables.

Grade 2.—Number work correlated with other subjects. Addition facts emphasized and in many places the multiplication table begun. Variations: In a few schools there is no number work; in some, at the other extreme, division is taught.

Grade 3.—The processes of addition and subtraction mastered together with some work on the multiplication tables, the tables often being completed. Variations: A few schools give no work at all, while some give considerable work in fractions.

Grade 4.—Multiplication and division mastered. Variations: Fractions are taken up in many schools.

Grade 5.—Fractions mastered, some decimals introduced, denominate numbers employed.

Grade 6.—Decimals as related to common fractions, with much problem work. In some schools simple interest and percentage are begun.

Grade 7.—Percentage and some of its applications.

Grade 8.—Business applications of percentage. Mensuration of solids. Variations: No arithmetic at all in the whole or the latter

half of the grade; the time devoted to algebra; algebra combined with arithmetic.

This general information above set forth is here represented in graphic form. In this form an opportunity is given to see when the study of certain topics is commenced, where the bulk of it is done, and where it is completed. This graphic presentation of the facts is based upon an investigation made of the courses of study in 28 typical cities. Aside from the subjects taught in all of the schools, it is interesting to note that compound interest is taught in 64 per cent of the schools, partial payments in 60 per cent, exchange in 43 per cent, partnership in 57 per cent, stocks and bonds in 71 per cent, compound proportion in 43 per cent, cube root in 28 per cent, the metric system in 36 per cent, algebra in 36 per cent, constructional geometry in 28 per cent, and graphical work in 7 per cent.

While it is understood that the number of school courses carefully examined was very small, these results indicate certain facts. First, certain subjects, namely, certain of those mentioned in the preceding paragraph, are slowly being dropped from the course, or else, in the case of the last four, are very slowly entering the course; second, in view of the fact that the courses examined were from larger cities, so that these results represent conditions above the average, it is evident that such subjects as cube root, partnership, exchange, compound proportion, and partial payments, are not being dropped as rapidly as they should be, while the other topics are not being introduced as rapidly as some discussions might lead us to think they are, regardless of whether they should be or not.

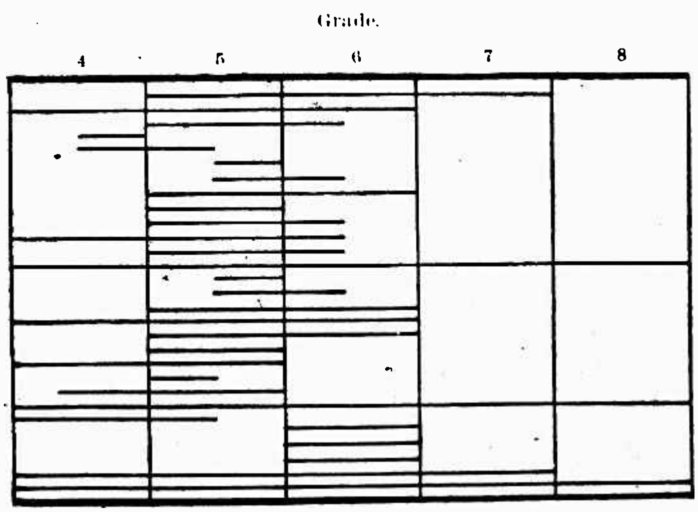

FIG. 1.—Grades in which common fractions are given; taught in all schools; most commonly emphasized in grade 5.

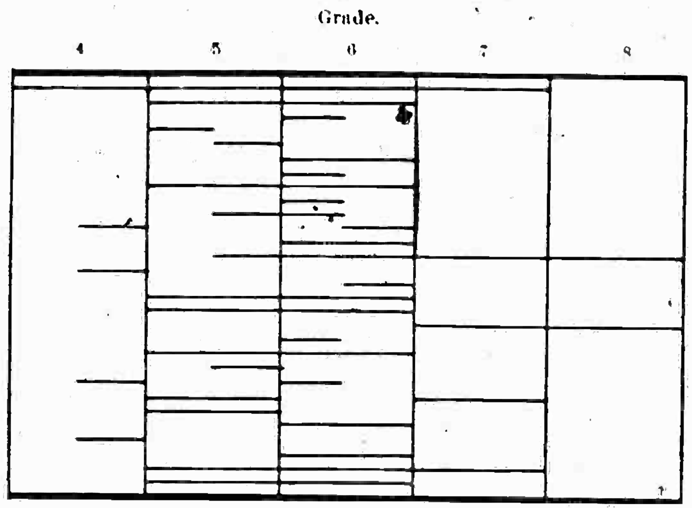

FIG. 2.—Grades in which decimals are given; taught in all schools; most commonly emphasized in grade 6.

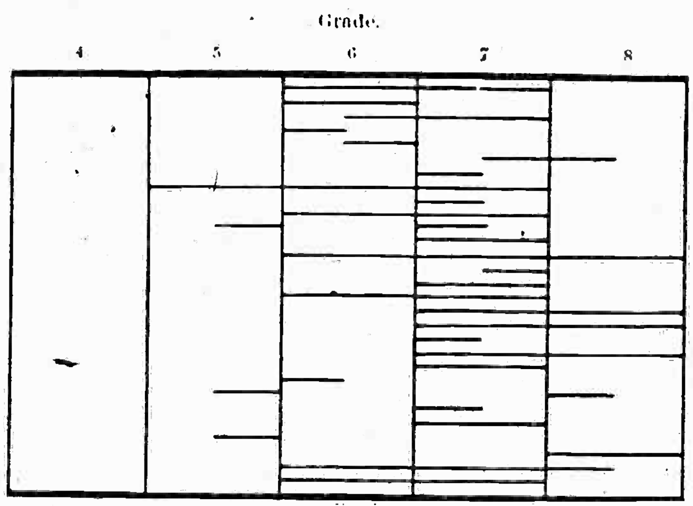

FIG. 3.—Grades in which percentage is given; taught in all schools; most commonly emphasized in grade 7.

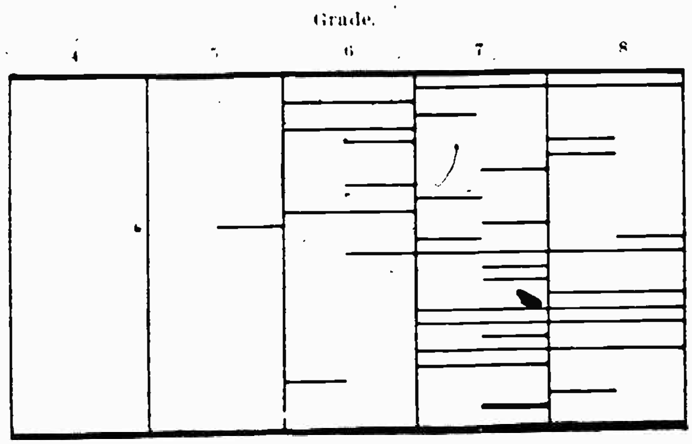

FIG. 4.—Grades in which simple interest is given; taught in all schools; most commonly emphasized in grade 7.

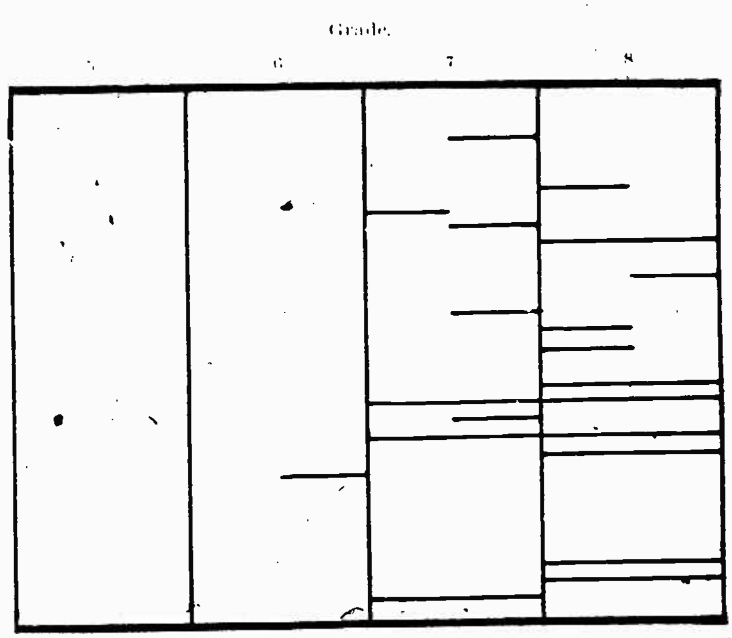

FIG. 5.—Compound interest; taught in eighteen out of twenty-eight schools, or 64.2 per cent of them; most commonly emphasized in grade 8.

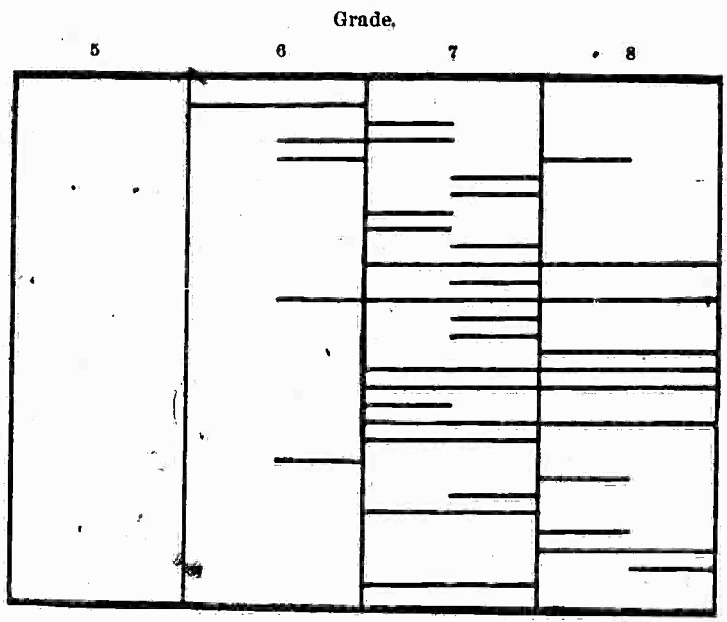

FIG. 6.—Commission; taught in most schools; most commonly emphasized in grade 7.

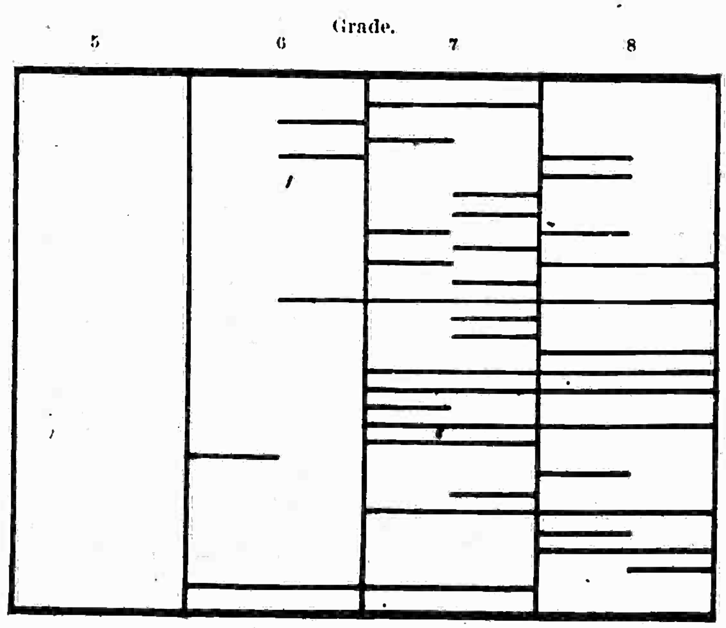

FIG. 7.—Commercial discount; taught in most schools; most commonly emphasized in grade 7 or 8.

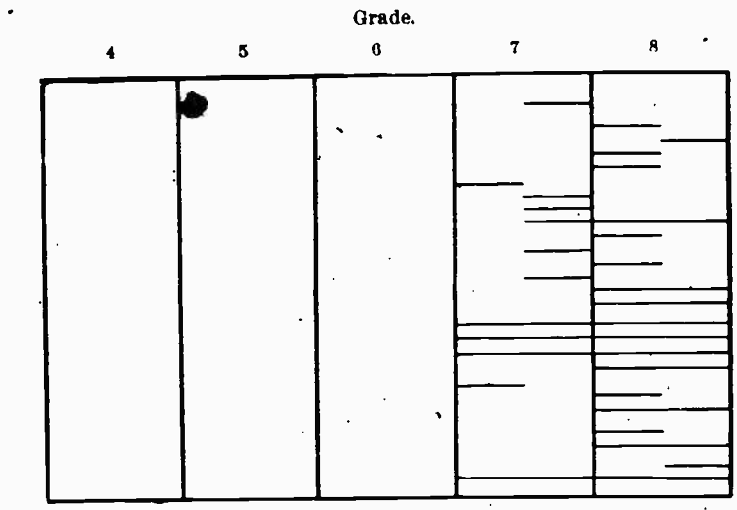

FIG. 8.—Bank discount; taught in most schools; most commonly emphasized in grade 8.

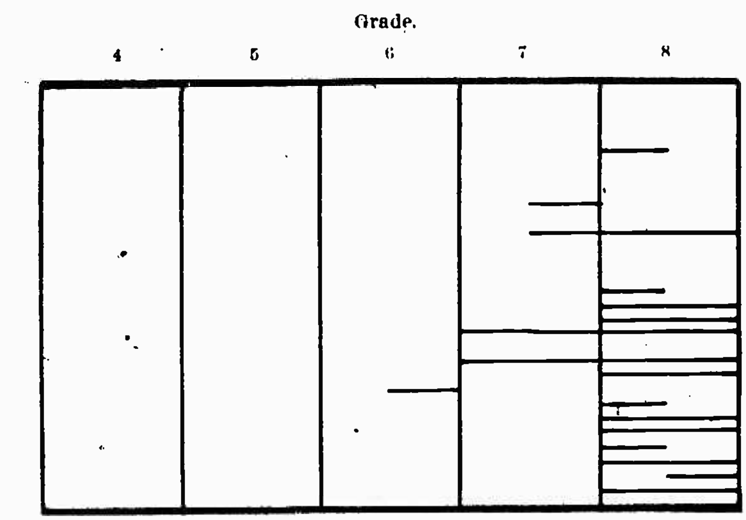

FIG. 9.—Partial payments; taught in 60.7 per cent of the twenty-eight schools; most commonly emphasized in grade 8.

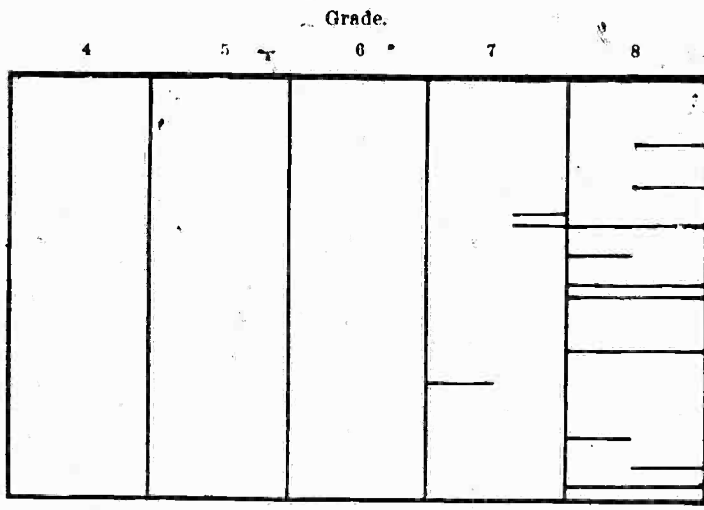

FIG. 10.—Exchange; taught in 42.8 per cent of the twenty-eight schools; most commonly emphasized in grade 8.

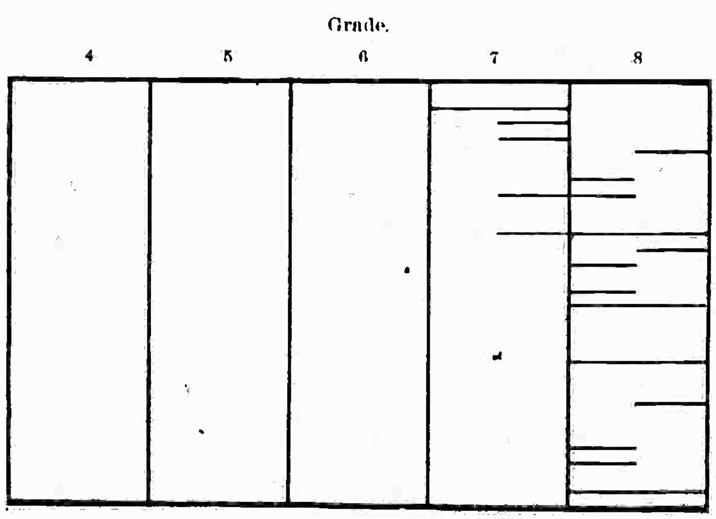

FIG. 11.—Partnership; taught in 57.1 per cent of the twenty-eight schools; most commonly emphasized in grade 8.

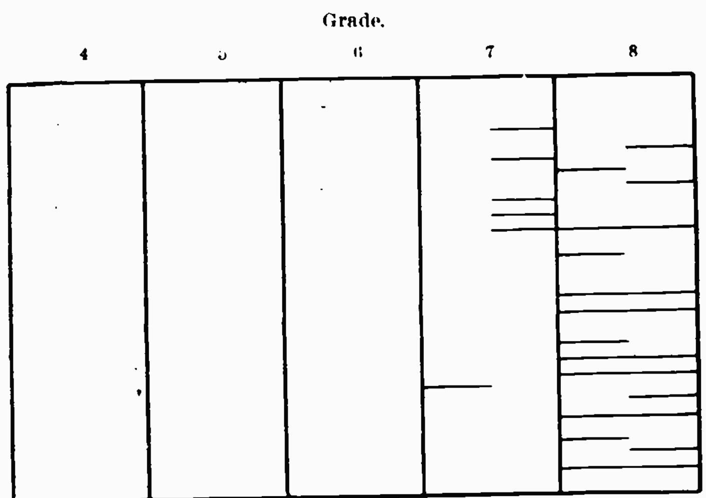

FIG. 12.—Stocks and bonds; taught in 71.4 per cent of the twenty-eight schools; most commonly emphasized in grade 8.

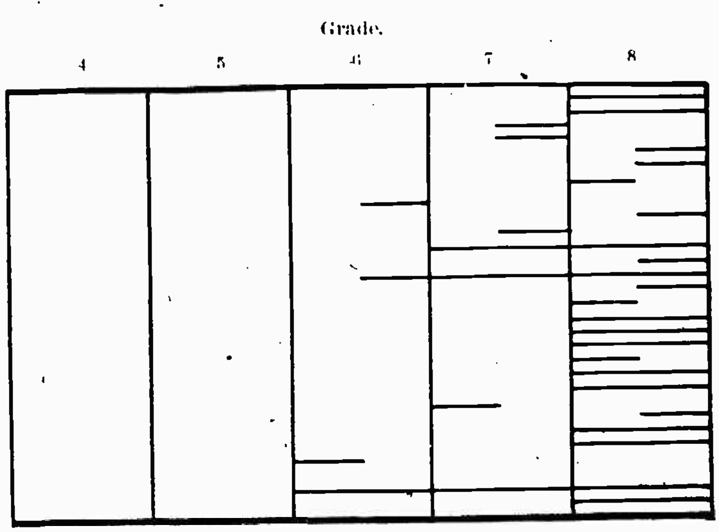

FIG. 13.—Simple proportion; taught in most schools; most commonly emphasized in grade 8.

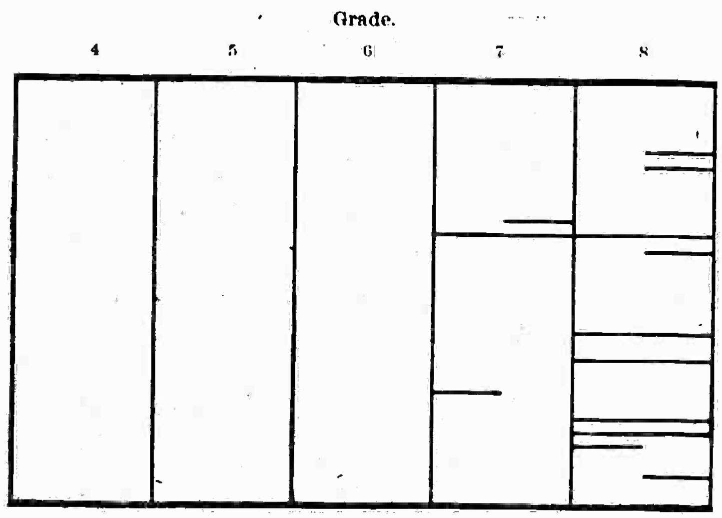

FIG. 14.—Compound proportion; taught in 42.8 per cent of the twenty-eight schools; most commonly emphasized in grade 8.

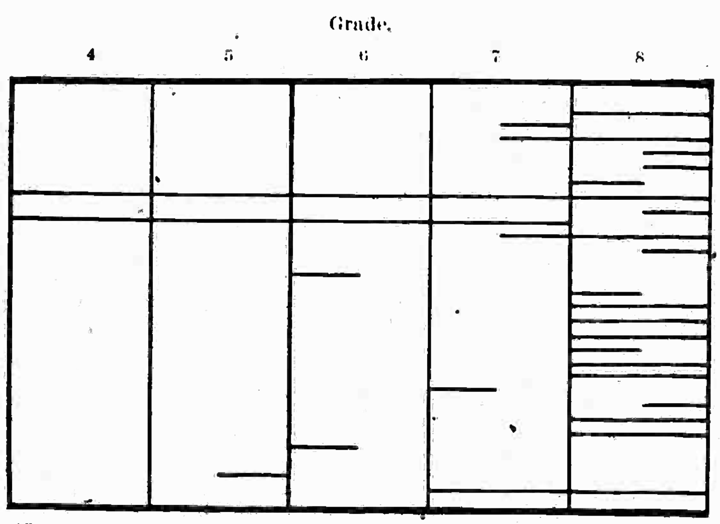

FIG. 15.—Ratio; the idea of ratio taught in most schools; usually emphasized in grade 8.

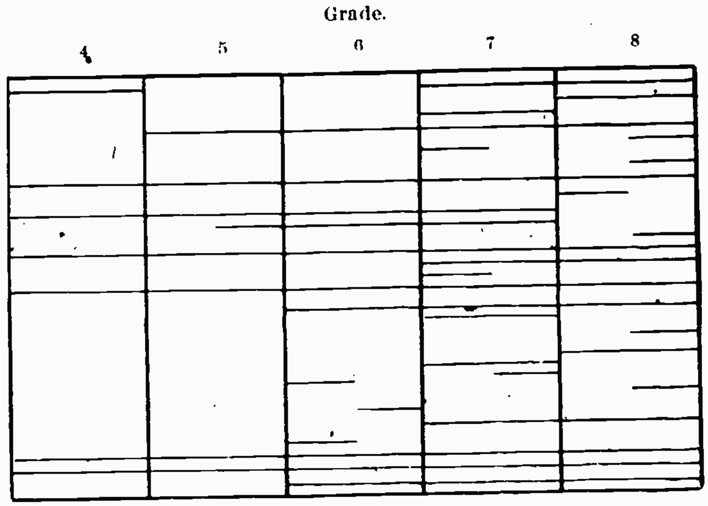

FIG. 16.—Mensuration of plane figures; taught in all schools; being spread out over the grades.

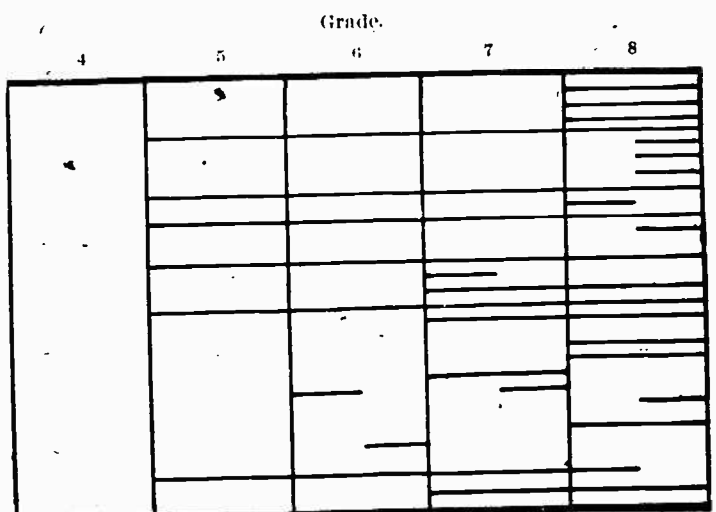

FIG. 17.—Mensuration of solids; taught in most schools; most commonly emphasized in grade 8.

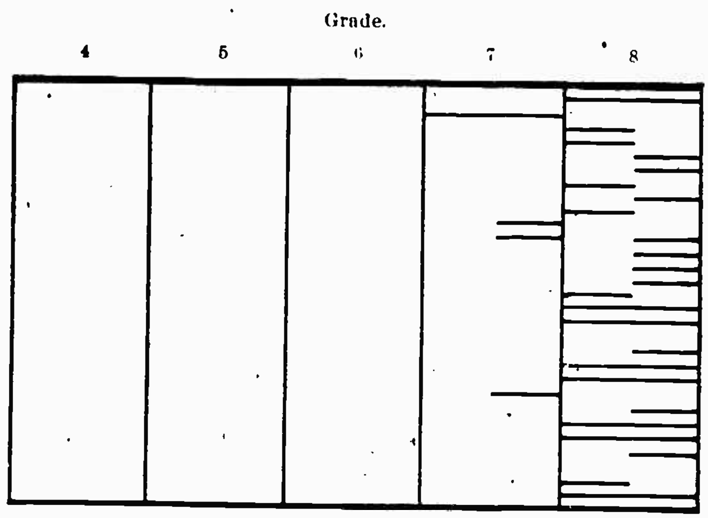

FIG. 18.—Square root; taught in most schools; usually in grade 8.

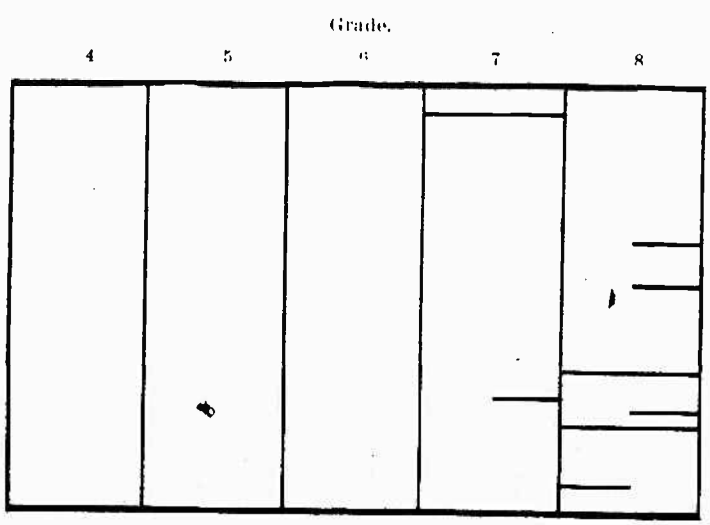

FIG. 19.—Cube root; taught in 28.5 per cent of the twenty-eight schools; usually in grade 8.

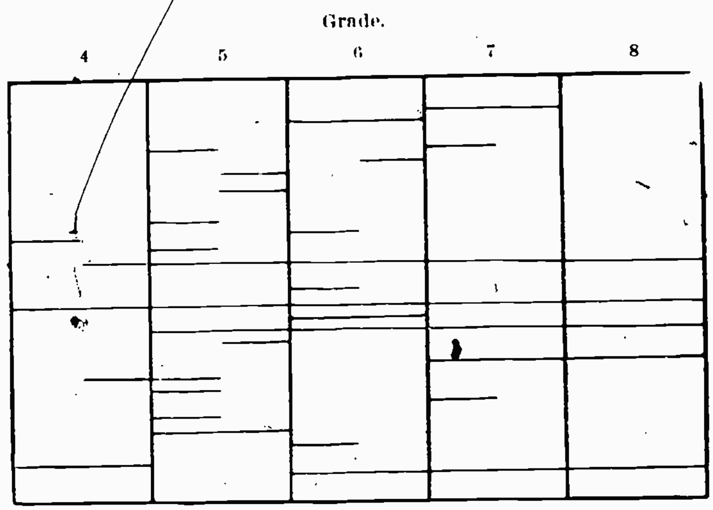

FIG. 20.—Factoring; taught in most schools; most commonly emphasized in grade 5 or 6.

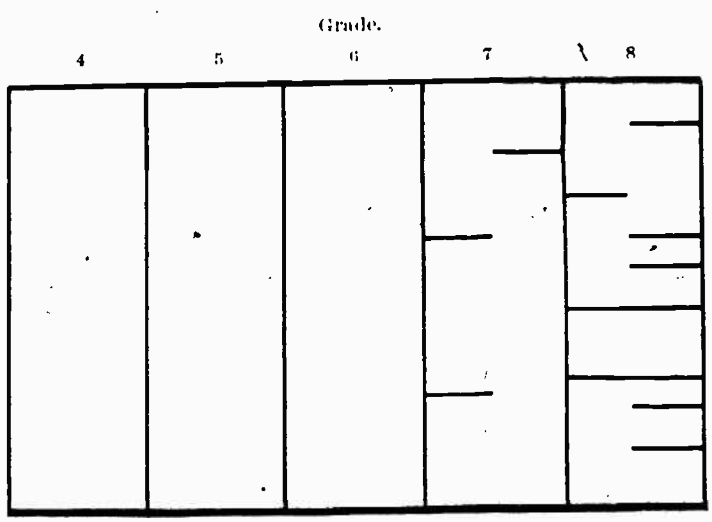

FIG. 21.—Metric system; taught in 35.7 per cent of the twenty-eight schools; usually in grade 8.

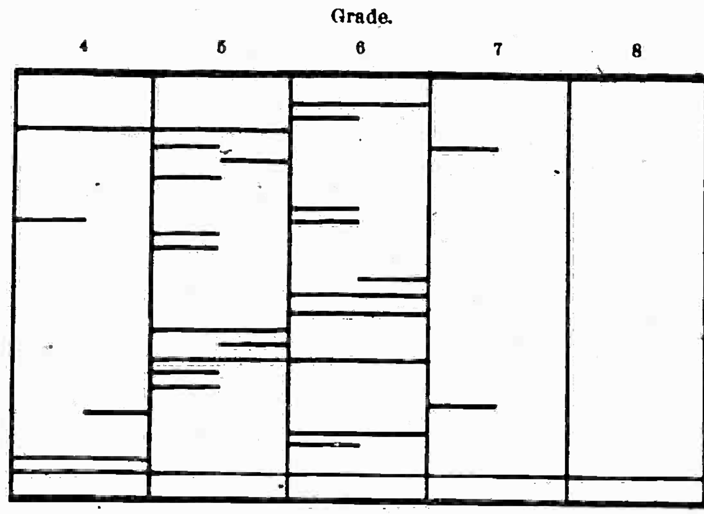

FIG. 22.—Least common multiple; taught in most schools; usually in grade 5 or 6.

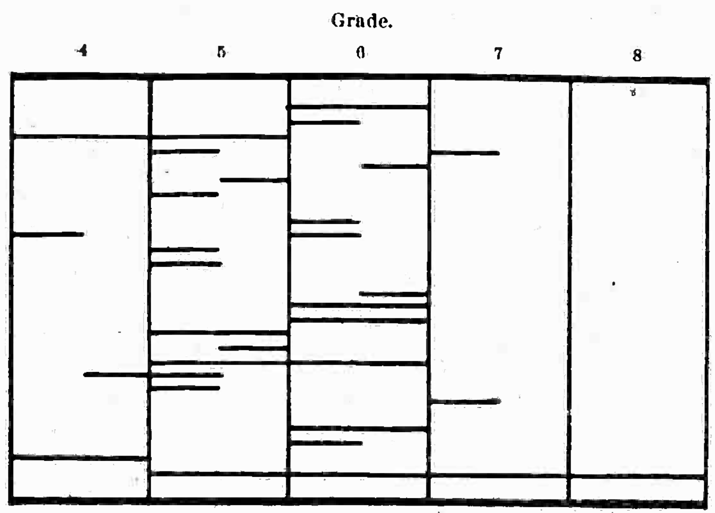

FIG. 23.—Highest common divisor; taught in most schools; usually in grade 5 or 6.

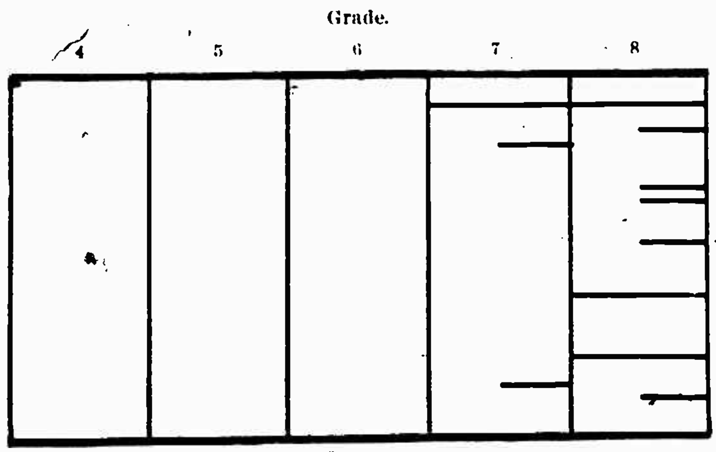

FIG. 24.—Algebra; taught in 35.7 per cent of the twenty-eight schools; usually in grade 8.

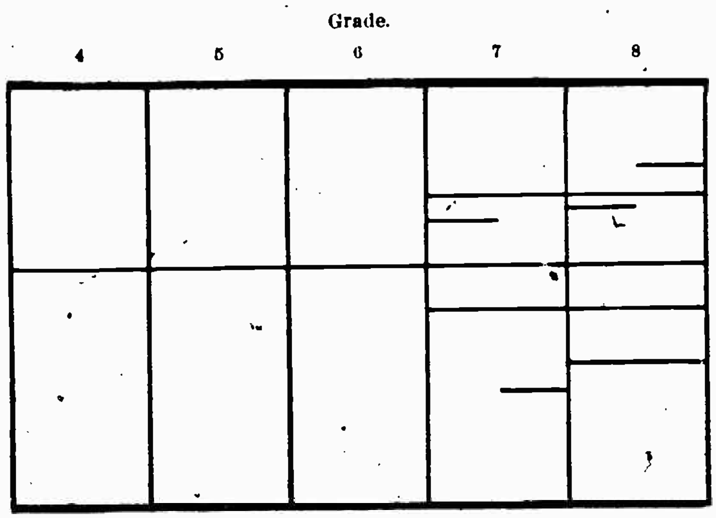

FIG. 25. Constructional geometry; taught in 28.5 per cent of the twenty-eight schools; usually in grade 7 or 8.

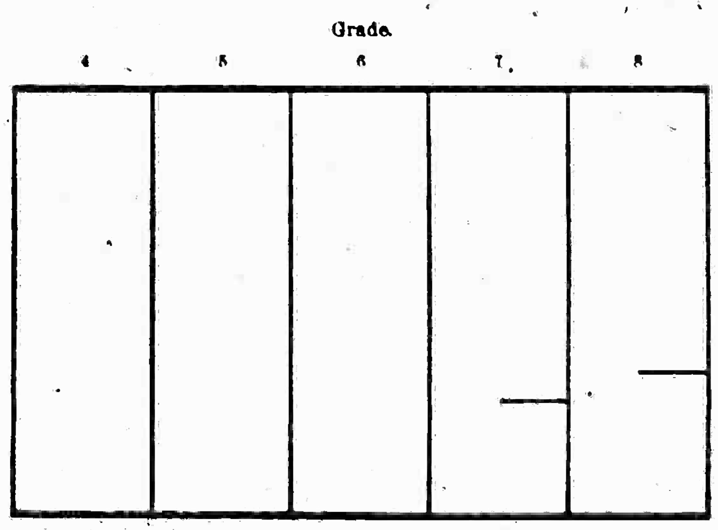

FIG. 26.—Graphical work: taught in 7 per cent of the twenty-eight schools.

TENDENCIES AS TO CONTENT OF COURSES.

Before considering the tendencies in the matter of the content of the curriculum, it would be helpful to consider carefully certain influences affecting the whole course of study in the elementary school. In order to economize space, however, a mere statement of these influences will be made. There is, among school men, and in the public to-day, an unwillingness to accept the doctrine of formal discipline as it was formerly stated, so that it is no longer possible to justify the retention of any topic or subject by claiming for it merely great disciplinary value; there is growing belief in the doctrine of interest as essential to will training, so that the possible attitude of the pupil toward a subject is taken into account more than formerly; there is greater and greater insistence upon the fact that the elementary school has as its function that defined briefly above, rather than, for example, preparation for the high school. All these influences would bear considerable discussion, for their effect upon the mathematics curriculum is very marked. For example, there is great pressure to simplify the course. This is being done by using smaller numbers in the work in arithmetic; by eliminating topics that are unduly confusing; by giving carefully graded simple problems; and by cutting down the extent and increasing the emphasis on the part that remains. There is also great pressure to modernize the course. This is being done by omitting obsolete problem material, topics, and processes, and by substituting therefor modern problem material. There is pressure to make the subject possess informational value as well as disciplinary value. This is

being done by a change in the problem material, such as introducing problems about our national resources and problems of local interest.

DISTRIBUTION OF THE MATERIAL THROUGH THE GRADES.

There are two general modes of organizing the whole course of study which have received much support. One is known as the topical plan, according to which each topic is treated once within the course of the eight grades, after which it is to be retained by the pupils. The other plan is known as the spiral plan, according to which each topic is touched upon repeatedly and more or less frequently throughout the course, so as to provide the necessary reviews. In its extreme form this plan may become highly confusing to teacher and class and may lead to a lack of sufficient concentration. The modern tendency seems to be to recognize and take advantage of the good in both plans; to adopt the spirit of the topical plan, to the extent that at various stages of the course certain mathematical ideas are stressed and certain number facts learned; and to adopt the spirit of the spiral plan, to the extent that there is frequent review of the essentials, so that the inevitable tendency of the pupils to forget is counteracted.

TYPICAL COURSES OF STUDY.

Further discussion of the content of the course might be given. As a matter of fact, there is only one satisfactory way to get a good idea of the content of any course and that is to read the course itself. This same remark might be made in regard to the subject which will be discussed in the next chapter, the matter of method in teaching mathematics, the manner in which the schools seek to influence method also being understood best by examining a course of study. Some courses of study are given in Part VI of this report.

III. EXAMINATIONS FROM THE POINT OF VIEW OF THE SCHOOL.

KINDS OF EXAMINATIONS CONSIDERED.

Although the American commissioners have assigned the subject of examinations to another committee, it seems proper to mention it briefly in this connection.

Examinations given in the school are of three kinds: (*a*) Those given by the teachers themselves; (*b*) those given by the supervising officers of the schools; (*c*) those given by a school to determine the qualifications of pupils proposing to enter the school.

The examinations given by the teacher are of two kinds; the ordinary "written recitations" and the stated examinations which may be required by school regulations. The first should not be called examinations in one sense of the word; they cover usually a short period

of previous instruction, and they are given as a means of affording the same sort of drill to all members of the class or as a means of detecting weaknesses as a basis for further instruction. Such written lessons are left entirely to the teacher, although it is advised that they be given frequently. Such examinations are in every respect desirable when conducted by the teacher with due regard for the general welfare of the pupils.

The teachers are sometimes required to give their pupils a written examination monthly, quarterly, or at some other stated times, and to use the marks obtained by the pupils for the purpose of determining their fitness for promotion. These examinations are more formidable in character; they cover a greater amount of previous instruction; and they are designed to test the pupils' power. Because of their consequences, such tests are feared by the pupils; in many cases they are considered unfair, and the result upon the pupils is a nervous strain.

The use of these two kinds of examinations is very general, and in both types the teacher is usually permitted to make out the questions. The practice of giving such examinations at the close of the year is decreasing. These tests are clearly given as an aid in determining promotion of the pupils, and it is against this type of examination that considerable criticism has been directed on account of its effect upon the pupils. In these examinations, however, the teacher usually has the power to adjust the test to the capability of her class, and in reading the papers she can allow for individual peculiarities.

The examinations given by the supervisory staff are primarily considered an administrative expedient. The questions are prepared by the superintendent, or by a committee of principals working under the direction of the superintendent, and they are then used in all of the schools of the system. The teachers have only in rare instances the right to add to or to change the questions before submitting them to their classes. The teachers mark the papers and then usually submit the results to their principals. These examinations are given for the purpose of setting standards of work throughout the system, interpreting the course of study, promoting uniformity, discovering weak points in the teaching, and revealing the general conditions in the school. In a broad way they are designed to benefit the pupils only indirectly, since the emphasis is placed upon the administrative advantages. The results may be used occasionally in determining a pupil's fitness for promotion, although it is seldom that failure in the examination alone would retard a pupil's progress. As an administrative device these examinations are effective. Their success depends upon the wisdom, the experience, and the ideals of the supervising officers. As a rule the examinations do not meet with favor among the teachers, who feel that the test

is not so much intended for their pupils as for themselves. From what has been said it is apparent that this is true to a large extent, although this depends upon the supervising force. Many of the teachers, recognizing the advantages gained through the examinations and having high professional ideals, do not take this view, however, and are willing to have their work compared with that of their colleagues. The examinations are opposed also because of their effect upon the pupils. It is contended that the pupils are subjected to a severe nervous strain, so that they do not do either themselves or their teachers credit. This is possibly true to an extent, although the evil results may be greatly lessened under wise supervision. Another objection made to such examinations is that they hamper the teachers in their work, so that there is little progress from year to year. This again depends upon the character of the supervision. Another objection made to such examinations is that fession works up to the supervising positions, and this element may be counted upon to promote rather than to hinder real progress in the schools.

REASONS FOR GIVING EXAMINATIONS.

Examinations have been ranked highly for reasons that vary according to the school employing them. Some feel that the value lies in the "mark" obtained by the pupil, this mark being sometimes counted as one-half of the pupil's standing and at other times counting as only a small per cent toward his rank in class or his readiness for promotion. Others deem that the practice in organizing quickly, in thinking clearly, and in stating accurately is the main value of examinations, a theory that tends to the use of monthly or biweekly examinations. Other schools find the worth of examinations to lie not in the benefit to the pupils (save indirectly), but in the help given to the teacher in the opportunities to check objectively her subjective ranking of pupils and of class progress. In the latter case the examination papers may not be returned to the pupils, the teacher noting the errors on which to work with both class and individuals. The other prominent reason for giving the examinations, the administrative advantages, has already been discussed.

TENDENCY IN THE USE OF EXAMINATIONS.

The present tendency in the use of examinations is to rank them as valuable but to let the value and the use of the papers be determined by the aim. When the pupils are to be benefited directly by practice (1) in separating essentials from nonessentials, (2) in clarifying a viewpoint through the enforced expression of it, (3) in learning to interpret the printed or written page independently, or

(4) in being tested on memory or power, then tests should be frequent, they should be carefully looked over by the teacher, and soon thereafter they should be made the basis of further instruction. Tests to further the first three of these ends are being given increasingly. Those tests or examinations of which the purpose is to help the teacher check her judgment of the pupils and test the portions of subject matter taught as to their relative value in giving the pupils ideals of and practice in perseverance, honesty, independent thought power, etc., need not be returned to the pupils. The teacher will note in this case the errors and will attempt to classify them; for example, as errors due to ignorance or to carelessness, those important or relatively unimportant, and class or individual errors. Such examinations are a form, and a very desirable form, of recitation.

STANDARDIZED TESTS.

Another view of the question of examinations concerned itself with "standardized tests." An investigation by questionnaire sent to a large number of schools showed that these tests are little known or used. These tests are discussed especially in a book, Arithmetical Abilities, by Dr. C. W. Stone, and published by Teachers College, Columbia University, New York. This book discusses certain tests given with extreme care in 26 school systems. The replies were carefully graded, the results examined, and deductions drawn. As a result these same tests can be used as a standard in other systems, giving an opportunity to compare results with those obtained in the 26 systems. Some schools and systems of schools have done this.

These tests may also be used to give a somewhat definite means of comparing system with system and to show the effects of changes in curriculum or in method of instruction, and doubtless an acquaintance with them would indicate other uses to which they could be put.

IV. METHOD OF INSTRUCTION IN ARITHMETIC.

By method of instruction in arithmetic is meant the special means of presenting certain knowledge of arithmetic to children in a way which is adapted to the immaturity of their minds. Such a method of necessity involves consideration of the subject matter selected for the course, of the organization and distribution of the material, and finally of the manner of bringing it before the pupils. The difficulty of making any general statements which shall apply to the country at large is very great, for in a country with school conditions such as exist in the United States there are and always must be wide variations in all three respects. Some common characteristics may be pointed out as being typical of the practice in the country; these may be inferred, first, from the content and organization of the textbooks,

and, secondly, from the known practice in some of the larger or more progressive school systems. There are some reform tendencies also to which attention should be directed.

Certain influences affecting the work of all of the subjects of the elementary school have produced, and are producing, changes in the method of instruction in arithmetic. On the part of the schools themselves there has been a growing regard for the ideals of psychology and pedagogy, and an increasing effort to put these ideals into practice as these have become more clearly defined. The ideal of appealing to the interests of the pupils, the maxim of allowing the pupils to learn through their own activity, the falsity of the doctrine of formal discipline as formerly understood, the clearer understanding of the processes of perception and of apperception with the resulting theory of developmental or inductive teaching—these and other ideals have led to changes in the methods of instruction from within the schools. These changes have only started, however, and the present must be regarded as a transition period. On the other hand, there have been other changes brought about as much if not more under pressure from without the school. There has been a growing independence on the part of the public in the matter of educational opinion; the result has been less acceptance as gospel of the statements of the schoolmen, more questioning of the worth of each and all of the subjects taught, and more insistent demand that the schools endeavor to provide instruction which will harmonize with and prepare for the conditions of modern life. There is on all hands growing sympathy for children, as a result of which their burdens are being made lighter, and, in particular, their period of school life is being looked upon not so much as a period of discipline in preparation for a further life of unpleasant tasks, but rather as a portion of their life which they can and should be helped to enjoy. In response to this influence, which has met with ready reception within the schools, there has come a tendency to simplify the work as much as is possible.

Instruction in arithmetic in the past has been dominated largely by a logical ideal and a disciplinary aim; these influences can be observed in the subject matter, in the organization of it in the text-books and in the means employed in presenting it to the pupils. The criterion observed in the selection of material for the course has been "to what extent is this topic necessary to round out the development of arithmetic, as a mathematical topic"; topics were selected frequently because they had certain mathematical interest rather than because they met some actual need of the pupils in either their present or later life. Long lists of abstract problems of unnecessary difficulty and complexity were introduced regardless of the fact that they were much more involved than any met in actual practice by most people. The concrete problems usually followed the abstract, were equally

difficult, and seldom were designed to appeal to the interest of the pupils; up to within recent years the problems often involved matters which are entirely obsolete. This material was organized topically; in the first chapter was given a treatment of notation and numeration which was designed to meet the complete future needs of the pupils, containing such numbers as billions, and beyond. Then followed chapters which contained the treatment of the four fundamental operations for integers, and so on through the course. Logically this arrangement was a natural one, but psychologically it was open to criticism. In the presentation of the material the book was again permeated by the logical ideal. Processes were either given by a rule followed by an illustrative example and exercises for the pupils, or some abstract explanation of the rule was attempted. The former method was the more common. This was a period of direct instruction and of drill on the part of the teacher, with little effort to provide any basis of experience for the pupils upon which they might build their arithmetic.

These ideals of instruction have by no means entirely passed, but in most of the textbooks in use now there is evidence of response to the influences mentioned in a former paragraph. This is a time of transition from the extreme of these views of the past to the more rational methods of the present. In the meantime the schools have passed through a period in which the quest after better methods has led some to adopt certain well-known one-idea methods. As examples of these may be mentioned the Grube method, with its attempted complete study of one number before passing on to the next, and the ratio method, in which the complete development of arithmetic was made to yield to a special mathematical form.

Whereas in the past instruction in mathematics in the elementary grades has been dominated by logical ideals and a disciplinary aim, at the present the leading purpose is to make the instruction as useful in content and as pedagogical in form as the conditions of school work permit. In regard to its content, remark has been made in a previous paragraph of the tendency to simplify the course through the omission of unduly complicated problems and of topics which have mathematical rather than general interest; with these omissions has come a greater insistence upon the essentials of arithmetic and additional problems whose special function it is to give the pupils an insight into certain phases of the life of their own country. The mathematician may fear that this process of elimination will result in a lower standard of mathematics; the fear is groundless, however, for the effort to direct attention to a more complete mastery of the fundamentals can not fail to raise the standard of mathematical equipment of the pupils.

Reference has been made also to the general organization of the material of the course in a previous part of this report, so that only a brief statement will be necessary at this point. The modern texts in most common use employ a combination of the older topical method and of the extreme form of the spiral method. The text is usually divided into sections which contain sufficient material for one-half year or one year. Within these sections the material is arranged topically; each section, however, reviews some or all of the topics contained in the previous sections. In this manner opportunity for review is provided and a better gradation of the material is made possible.

The remainder of this section of the report will be devoted to a brief statement of some of the special means employed for bringing the arithmetic before the pupils.

In all schools instruction is given to groups of pupils varying in number from five in small communities to thirty or even more in some of the larger communities; in cities the average is about 25. As a rule the arithmetic class meets daily for from 20 to 30 minutes, in a period called the class or recitation period; the pupils usually have another period of about the same length for study in school, giving a total of from 200 to 300 minutes per week for arithmetic. Custom in regard to home study varies; the tendency is to cut down on home study in arithmetic, except possibly in the upper grades. Each pupil possesses a book, which is more than a mere collection of problems; in fact this book provides for many classes about all of the "explanation" or development that some of the pupils ever receive. In most cases, however, new topics are taken up in class by the teacher, who gives such development of the topic as seems satisfactory to her, after which the main responsibility of the pupils consists of the solution of similar problems in their study time. In the past it has been all too common for the class time to be occupied with indiscriminate recitation on this work done by the pupils in their study time; the present tendency is in the direction of use of this class period either for instruction on new topics or for vigorous drill upon past and present work under the direction of the teacher. Another characteristic of the class period is the rather uniform practice of devoting from one-third to one-half of the time to mental arithmetic, i. e., to computations without the aid of pencil and paper. The function of this type of work is to produce skill in mental computation, and, secondly, to maintain and increase efficiency in the portions of the subject taught previously, by having the pupils solve many simple problems.

In the actual presentation of new topics, custom varies from the unsatisfactory plan of direct instruction, of mere telling of the fact

or of the process and of drill upon it until the pupils know it, to the desirable form of presentation in which some capable teacher consciously and successfully develops the new idea inductively. One of the obvious effects of the influence of psychology upon method is the greater insistence upon inductive or developmental teaching. Unfortunately, the mass of teachers have an entirely inadequate idea of the underlying psychology and in fact of the essentials of this general mode of procedure in teaching a new topic. Most of the teachers are impressed with the need of teaching the simple before the complex, the concrete before the abstract, the special before the general; but clear-cut ideas of the philosophy of the practice and of the function of the definite stages which are possible in inductive teaching are usually lacking. This may be due to the fact that all of mathematics has been looked upon as a deductive science and that no special effort has been made to impress the idea of inductive teaching; it is probably so because little attention has been given to the problem of method as such. Progress in the direction of better understanding of the stages of mental development has been reflected in the more progressive schools in efforts to provide a teaching method in harmony with this knowledge.

The practice of using objective aids in presenting new topics is the most common result of the efforts to regard the teachings of psychology. The texts and the teachers endeavor to lay a foundation of experience for the pupils upon which they can build their mathematics, and belief in the efficacy of objective work in accomplishing this is quite general. The theory behind this belief is often the crude idea that objects help and that the younger children need this assistance more than do the older ones, with the result that objective aids have been used decreasingly from grade one to grade eight. The relation of objective work to inductive teaching has not always been clear; that the use of objects helps sometimes to present to the pupils clearly individual cases of some general idea which is being taught, out of which the pupils are to form the general idea by a process of generalization—this fact is not as widely understood as would be desirable. The result has been a certain amount of waste in the use of objective aids and consequent dissatisfaction with such work. There has been a tendency to use objects too much in the lower grades, and, on the other hand, there has been the fault of not using objects sufficiently in the upper grades. While this may seem to represent an inconsistent state of affairs, the explanation is easy; the objects have been used to excess at times with young children and then, for the reason mentioned above, when new topics have been undertaken in the upper grades, this means of aiding the older pupils has been neglected, with the result that the pupils have not always gotten clear

ideas. The great desideratum at the present time is that objects should be used more understandingly.

The material used in the schoolroom as objective aids is limited and highly artificial, consisting of tiles, pegs, splints, toothpicks, squares of cardboard, etc. This material has the special advantage of being adapted to the conditions of schoolroom work; it is inexpensive, compact, easily handled and not too attractive. Besides this material which can be handled, there is a growing use of geometrical figures, especially of rectangles; these can be quickly drawn, are easily changed and adapted to the conditions of particular problems, are divisible, in fact, lend themselves readily to the needs of the situation. The use of rectangles in teaching fractions is especially to be mentioned. In the selection of objects, two qualities which are particularly desirable, in fact, necessary in some cases, are not always realized and when realized are not always readily obtainable; these are the elements of naturalness and variety. The element of naturalness is probably sufficiently appreciated; in the early grades, the pupils themselves, the seats in the room, the material which is distributed in connection with the work of all the classes, the selection and grouping of the pupils for their games, these are some of the more natural sources of material for concrete expression of the number facts studied. Similar material for use in the upper grades is not always so readily obtained; of course, in connection with denominate numbers, the actual weights and measures are brought into the schoolrooms in the progressive schools. As much as possible, the problem material in all of the grades is selected from the experience of the pupils, and the actual conditions of some of the topics are reproduced in the schoolroom. For example, in teaching stocks and bonds, the class may resolve itself into a broker's office, one member of the class acting as the broker and the rest of the class acting as buyers or sellers of stocks and bonds; again, the class in a lower grade may resolve itself into a store with similar distribution of responsibilities among the members of the class; when discussing commercial paper and business forms, samples are brought into the classroom so that the pupils may get the feeling that there is a real connection between the work they are doing in the schoolroom and that of adults who are engaged in the work of the world. An effort is made to permit the pupils to handle the material themselves.

The other quality mentioned as desirable, namely, variety, is not appreciated as much as it should be. The advantage in having a variety of objects as a means of arousing interest is usually clear, but the necessity of variety in presenting particular cases of a general idea which is being developed inductively is not clearly understood. For example, in presenting the idea of tens, bundles of splints

containing 10 splints are often used; the pupil soon learns to associate the word ten with one of these bundles without really understanding that the bundle contains 10 splints. This difficulty is obviated somewhat by using a variety of means of representing 10. Other illustrations might be given to make the point that in objective teaching care must be taken to prevent too immediate association of the idea being taught with any particular manner of representing that idea in a concrete way.

By way of summary of this section, devoted to the discussion of methods of teaching arithmetic, it may be said that there has been great progress in establishing the principle that psychological rather than logical ideals should permeate the teaching of arithmetic, and that there has been some progress toward incorporating this principle in the actual teaching practice, in the form of objective work and inductive developments.

V. TRAINING AND QUALIFICATIONS OF TEACHERS.

SCOPE AND SOURCE OF MATERIAL.

Part V deals with the training and qualifications of teachers of mathematics in the elementary schools. It is based in large part on the report of subcommittee No. 3 of this general committee, whose field of study was limited to the training of teachers for work in grades 1 to 6. The committee appointed to study the same problem for grades 7 and 8 was unable to report up to the time when this summary was first made. That report has since been added to part (b) of this general report.

GENERAL TRAINING.

In an earlier paragraph the statement was made that in the country at large over 425,000 teachers were employed in 1906, and that about 100,000 new teachers are employed annually. Further statistics show that not more than 20,000 of these can receive pedagogical training annually as that is the limit of the capacity of professional training schools of all sorts in this country. At best, therefore, not more than one-fifth of the new teachers can have received any professional training whatever.

Generally, even in the districts where educational advantages are very meager, prospective teachers must be high-school graduates. Graduation from high school implies the completion of a course equivalent in content to the following: Three years of English, four of Latin, two and one-half of mathematics, two of science, two of history, and two years of additional subjects. In addition to this scholastic training the prospective teacher must attend a normal training school within the limits mentioned. In some States a

minimum length of time must be spent in such school. In most of the cities, in which, as has been said, more than one-half of the population is gathered, this normal training and even previous experience is absolutely insisted upon. It might be said at this point that many of the best teachers in the schools owe most of their training to the hard school of experience, especially hard when that experience consists of service in one of the rural schools discussed in chapter one. It may also be said that the larger cities seek recruits for their teaching staff very commonly from the graduates of this same school of experience.

TRAINING IN THE NORMAL SCHOOL.

The training in a normal school usually consists of general pedagogical theory, some special training in the methods of the different subjects including mathematics, some academic study of the subjects, such as algebra in mathematics, some observation of teaching, usually in all subjects, and more or less practice teaching.

From a study of the practice of the normal schools the following points of variability seem existent:

1. Method of teaching enters into "methods" courses from the extreme of being merely incidental to the other of occupying full time and attention.
2. Current literature on the teaching of mathematics is sometimes used extensively and sometimes not used at all.
3. Similarly with consideration of games and recreational devices in arithmetic; sometimes they are considered, and again not.
4. The course of study for the whole curriculum is sometimes not studied and is sometimes carefully analyzed.
5. The history of the development and of the teaching of mathematics is given similar varying emphasis.

RECOMMENDATIONS FOR A TEACHING COURSE.

1. A foundation in subject matter as a basis for the professional study of mathematics should include a minimum of one-half year of high-school arithmetic, one year of algebra, and one year of geometry.
2. Exclusive of all courses in psychology, pedagogy, principles of teaching, general method, and history of education, a minimum of one-half year of the professional study of arithmetic should be required to include the following:
   A. The teaching of elementary mathematics—"Special Method."
      (*a*) The special pedagogy of arithmetic.
         1. The more elementary phases of the psychology of number.
         2. Principles of general method applied to teaching arithmetic.

(a) Educational values of arithmetic and the place of arithmetic in the general educational scheme.

(b) The organization of the general elementary school curriculum in arithmetic.

(c) Organization of typical units of subject matter for presentation to appropriate grades.

(d) Development and writing of typical plans for teaching.

(e) The utilization of local and general economic studies for number applications.

(f) Observation and discussion of typical lessons in the grades showing concrete applications of the principles developed.

(g) The place of games and other recreational devices in grade number work.

B. The historical development of the teaching of arithmetic, and the place and value of certain "methods," such as those of Pestalozzi and Grube.

3. That every school engaged in the preparation of teachers of mathematics should develop a museum of materials, apparatus, books, pamphlets, papers, which will aid in interpreting such features as the historic development of the subject, the present-day practice, and the nature of the textbooks.

4. That the head of department of mathematics should be largely responsible for the organization of the course of study in mathematics in the training school, in cooperation with the department of education and the supervisor of the training school.

5. That the head of the department of mathematics should aid in the supervision of the teaching of mathematics in the training school.

6. That the head of the department of mathematics, as well as the critic teacher, should give demonstration lessons in the training school, illustrating the principles of teaching, developed in the methods class.

7. That the points of emphasis in all observations, discussions, written plans, and criticisms should be upon the basis of fundamental principles rather than upon devices or petty details.

VI. TYPICAL COURSES OF STUDY.

INTRODUCTION.

It is almost certain that much that has been given in this report will be intelligible only if interpreted. Since the main interest will probably center on the curriculum itself and upon the teaching helps given to the teachers and recommended for their use, it seems

worth while to include in this report two typical courses of study. The first is the course of study recently issued for the guidance of the teacher of the rural schools of one of our States, but generally considered too difficult in the first two years. The second course has recently been prepared for the schools of one of our cities.

A STATE COURSE OF STUDY.[1]

SUGGESTIONS.

The work in arithmetic should, first of all, produce accuracy and rapidity in computation. Accuracy can be assured only by holding the pupil to exactly correct results from the beginning. Pupils should be held to correct results and made to detect and correct even the slightest error.

Rapidity of computation can be secured only through much practice and drill. Throughout the first three years of the work the entire time in the subject should be given to securing this accurate and rapid work in the fundamental operations.

While drill in the facts of number during the early grades should be largely in the abstract, with few if any problems, it is suggested that some concrete work in these combinations be given. Illustration: 5 dollars and 4 dollars are how many dollars?

FIRST YEAR.

(FIRST HALF.)

Count numbers to 100. Read numbers to 100. Write numbers to 100.

Memorize the 20 of the 45 combinations in addition, the sum of which does not exceed 9.

Give plenty of oral drill together with seat work and blackboard work like the following:

2	5	3	4	2	7		2	1	2	3	2	3	7	5	1	2	6
4	2	2	4	4	1		6	8	5	2	3	4	2	3	7	3	2

and have pupils get correct results by copying, where necessary, the results from the combinations placed upon the blackboard. This work together with oral drill and tests will in a short time fix these combinations in mind without the use of objects and the consequent formation of the pernicious habit of counting the fingers in adding.

From the first, drill in these combinations should be given in such manner as to prepare for subtraction as well as addition.

Illustration.

Teacher—Five-four?
Pupil—Nine.
Teacher—Five and what are nine?
Pupil—Five and four are nine.
Teacher—Four and what are nine?
Pupil—Four and five are nine.

[1] Prepared by the Education Department of New York State. As here given the course is somewhat abridged.

When the combinations are learned in this manner, the work in subtraction, if taught by the Austrian method (sometimes called the addition method), is learned at the same time as addition.

Illustration.

$$\begin{array}{r} 87639 \\ -42316 \\ \hline \end{array}$$

Method.

6 and 3 are 9.
1 and 2 are 3.
3 and 3 are 6.
2 and 5 are 7.
4 and 4 are 8.

At the close of the first half year pupils should be able to count, read, and write numbers to 100. They should know the 20 combinations, the sum of which does not exceed 9 and their use in addition.

Objects should be used only for consecutive counting and developing the idea of number in the abstract. Children should never be taught to count two groups of objects to find the sum. This gives the idea that addition is counting, which is a serious hindrance to accuracy and rapidity in work. Nothing has done more injury in number work than the too long continued and injudicious use of objects in its teaching.

SECOND HALF.

Continued drill in addition and subtraction with the 20 combinations learned in the first half.

Count to 100 by twos, by fives, by tens.

Drill in adding columns of figures on board and cards arranged for this purpose, the sum not to exceed nine.

Memorize the remaining 25 combinations in addition.

Give oral drill, seat, and blackboard work in plenty with examples like the following:

$$\begin{array}{r} 6845324 \\ 4386978 \\ \hline \end{array} \qquad \begin{array}{r} 869345 \\ 124588 \\ \hline \end{array}$$

Children are here taught to carry in addition.

Method. Teach the very best model, insist upon its exact imitation and much repetition and drill. Make no attempt to explain the process. This is the time to teach the art of computation, not the science of numbers.

SECOND YEAR.

(FIRST HALF.)

Continued drill in the use of the 45 combinations in addition and subtraction. Drill on series work in addition.

Illustration.

$$\begin{array}{cccccc} 2 & 12 & 22 & 32 & 42 & 52 \\ 5 & 5 & 5 & 5 & 5 & 5 \\ \hline \end{array}$$

Continue drill by counting by twos, threes, fours, fives.

Illustration.

Count to 50 by twos beginning with 0, beginning with one.

Count to 50 by threes, beginning with 0, beginning with 1, beginning with 2.

Count to 50 by fours, beginning with 0, beginning with 1, beginning with 2, beginning with 3, etc.

Drill in subtraction—Austrian method.

Illustration.

1235	6 and 9 are 15.
— 786	9 and 4 are 13.
449	8 and 4 are 12.

The entire time of this half year should be given to use and drill of the facts of number learned as here specified. By the close of the period simple numbers will be added and subtracted with accuracy and facility and much progress made in the addition of columns of figures.

SECOND HALF.

Notation and numeration of numbers through first three periods.

Continued drill in addition and subtraction, especially in the addition of columns.

Memorize the 45 combinations in multiplication.

Teach these combinations so that preparation is given for division at the same time that multiplication is being taught, that is, have the pupil answer the questions, how many sixes in 24 and how many fours in 24 as well as to state that 4 times 6 are 24.

Give much oral drill, seat work, and board work of the following character:

3 2 6 4 5 3	8 6 2 4 5	2 0	3 0	4 0	5 0	6 0
2	3	2	3	4	5	6

Teach the process of carrying in multiplication.

(1) A good model. (2) Imitation of the model. (3) Repetition, drill. (4) No explanation.

THIRD YEAR.

FIRST HALF.

Continue drill in counting. Count by fives to 100, beginning with 0, beginning with 1, beginning with 2, beginning with 3, beginning with 4. Count by sixes, beginning with each of the numbers from 0 to 5, inclusive.

Short division with 2, 3, 4, 5, 6, 7, 8, and 9 as divisors. Give thorough and systematic oral drill as a preparation for this work. Before asking a child to use 2 as a divisor, he should be able to tell instantly how many times, with remainder, 2 is contained in every number to 20. When 3 is used as a divisor, a similar drill should be given in numbers to 30, with 4 to 40, with 5 to 50, with 6 to 60, with 7 to 70, with 8 to 80, with 9 to 90.

Multiplication with two or more figures in the multiplier.

Definitions of terms used in fundamental operations. Addend, sum, minuend, subtrahend, remainder, multiplicand, multiplier, product, dividend, divisor, quotient.

Continue drill of all previous work.

Measures—areas of simple figures with use of ruler correlating with drawing. Dimensions used inch and foot. Square inch and square foot, and the fractions $\frac{1}{2}$ and $\frac{1}{4}$ as applied in the use of the linear unit used in measuring.

SECOND HALF.

Long division. Never allow a child to use long division when the divisor is a single figure.

Multiplication tables of the tens, elevens, and twelves and their use as divisors in short division.

Begin long division with easy divisors of two figures. Remember that 31, 41, 51, 61, 71, 81, and 91 are easier divisors than 14, 15, 16, 17, 18, and 19, 27, 37, 58, 76, etc.

Give much oral drill and written work for accuracy and rapidity in all of the fundamental operations.

Tests for divisability of numbers by 2, 3, 5, 9, 10.

Definition of factor, prime factor. Memorize prime factors up to 25 (1, 2, 3, 5, 7, 11, 13, 17, 19, 23).

Oral drill in finding prime factors to 100.

Continue measurement of objects in schoolroom for surface measurement. Additional measure: The linear yard. Learn liquid measure of pint, quart, gallon.

At the close of this year's work the pupils should be able to add, subtract, multiply, and divide numbers with accuracy and facility.

FOURTH YEAR.

FIRST HALF.

Roman numerals I to C and by hundreds to M.

United States money. Writing and reading of numbers expressing dollars and cents. The use of this knowledge in problem work demands its introduction at this time.

Simple problems, oral and written, connected with daily life. Original problem work by the children. Problems in bills and accounts, especially where one of the factors is 12 or less the extension of the products making up the sum to be done mentally.

Teach cancellation. Have all problems stated before being worked. Use cancelation whenever possible in the solution of these statements.

Dry measure: Pint, quart, peck, bushel. Problems in reduction only.

Simple fractions and equivalents. Develop objectively. Also teach idea that a fraction is an indicated division with the dividend above the line and the divisor below it.

Fix the principles: (1) Multiplying the numerator or dividing the denominator of a fraction by any number, multiplies the fraction by that number. (2) Dividing the numerator or multiplying the denominator of a fraction by any number divides the fraction by that number. (3) Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.

Continue drill throughout the year in the fundamental operations. Facility in these operations demands that this work be continued daily throughout the entire course.

Changing fractions to equivalent fractions of higher and lower demoninations.

Give much oral drill in changing fractions to equivalent fractions of higher and lower denominations and in reducing them to a common denominator. Denominators of these fractions should not exceed 24.

Make little use of denominators 13, 17, 19, 29, and 39.

Addition and subtraction of fractions, the denominators of which do not contain more than two figures.

SECOND HALF.

Addition and subtraction of mixed numbers involving fractions already used in the manner here indicated. (Mixed numbers should never be reduced to improper fractions in addition and subtraction.)

Least common multiple of numbers to 100 and application in reducing fractions to a common denominator. (Use method of factoring in equation form.)

$$45 = 5 \times 3 \times 3 \qquad 5 \times 2 \times 3 \times 3 = \text{L. C. M.}$$
$$30 = 2 \times 3 \times 5 \qquad 5 \times 3 = \text{G. C. D.}$$

Multiplication of—

1. A fraction by an integer.
2. An integer by a fraction.
3. A fraction by a fraction. Use cancellation in all cases where possible.

Written work in preceding cases should be preceded by much oral drill with simple numbers.

Division of—

1. A fraction by an integer.
2. An integer by a fraction.
3. A fraction by a fraction. Use cancellation whenever possible.

Have the written work preceded by much oral drill, as in multiplication of fractions.

Continue the solution of problems involving the principles and processes learned in all previous work.

Problems to be stated before being solved, including oral analysis.

Cubic measure. Volume involving cubic inches, cubic feet, and cubic yards.

FIFTH YEAR.

FIRST HALF.

Reading and writing of decimals.

Reduction of common fractions to decimal fractions and decimal fractions to common fractions.

Fundamental operations in decimal fractions. (The Austrian method of placing the decimal point in division is recommended.)

Memorize the aliquot parts of \$1 or of \$100.

Review tables of linear measure, square measure, and cubic measure.

Memorize number of cubic inches in a bushel and in a liquid gallon.

Reduction ascending and descending as applied to these tables.

Application of square measure to finding area of squares, triangles, rectangles, and to problems in painting, carpeting, and plastering.

Application of cubic measure to finding volume of rectangular solids, the capacity of bins and cisterns, cost of masonry.

(Teach accurate capacity by using 2150.42 cubic inches in a bushel; also approximate capacity by using one bushel equal to $\frac{5}{4}$ cubic feet. In liquid measure use 231 cubic inches to gallon, also $7\frac{1}{2}$ gallons = 1 cubic foot.)

SECOND HALF.

Memorize tables of avoirdupois weight, dry measure, liquid measure, English money, time, and circular measure. Also learn number of pounds in a bushel of potatoes, wheat, corn, barley, oats, barrel of flour, and apply in the solution of problems. Also learn value of franc, pound, and mark in United States money.

Reduction ascending and descending as applied to these tables.

Other tables in denominate number besides those memorized may be used in problems, but pupils should be allowed to refer to books for facts used.

Problems in the form of bills and accounts continued.

SIXTH YEAR.

FIRST HALF.

Review the work of fractions, with special emphasis on the three questions:

1. To find a fractional part of a number.
2. To find what fractional part one number is of another.
3. Given a fractional part of a number and its relation to the whole, to find the whole.

Illustration: $\frac{3}{5}$ of 35=? What fractional part of 35 is 21? 21 is $\frac{3}{5}$ of what number?

Review denominate numbers and drill on industrial problems demanding their use.

Fundamental operations in percentage:

1. Any per cent of any number. Aliquot per cents. Use the term per cent interchangeably with hundredths. Learn thoroughly the aliquot parts of a unit and use interchangeably with hundredths and per cent. (See table given in first half of fifth year.)
2. The per cent one number is of another.
3. One number is a given per cent of what number?

Applications to problems.

SECOND HALF.

1. Profit and loss, including (*a*) marking goods; (*b*) trade discount; (*c*) cash discount.
2. Commission—using only practical and common problems.
3. Simple interest.
   (*a*) To find interest on any sum of money for a given time at a given rate.
   (*b*) The making of promissory notes and the computing of interest thereon.
   (*c*) Problems in interest when three of the elements of P, R, T, and I are given to find the fourth.
4. The simple equation and the use of the unknown quantity x can be profitably used in the solution of some problems.

SEVENTH YEAR.

Pupils who have completed the work of the six preceding grades should be able (1) to read reasonably large numbers at sight and to write numbers rapidly from dictation; (2) to add problems 5 figures wide and 20 numbers deep ac-

curately at a fair rate of speed, i. e., in about two minutes; (3) to perform all fundamental processes in arithmetic rapidly and accurately; (4) to reason quickly and explain simple problems; (5) to handle ordinary fractions—common and decimal—without hesitation; and (6) to comprehend the fundamental principles of percentage and their applications.

At the end of the seventh year any one of the following courses may be pursued: (1) To follow syllabus in algebra for the eighth grade; (2) to give a part of the time to arithmetic and a part to algebra; (3) to give the entire time to arithmetic.

If the study of arithmetic is to stop with the end of the seventh year it will be advisable to follow the syllabus for this year and to include such topics of the eighth year, i. e., stocks and bonds, insurance and taxes, as are deemed necessary to round out the subject.

If the arithmetic is to continue through the first half of the eighth year, leaving the other half free for algebra, it will be necessary to select those topics in arithmetic which are considered essential to the laying of a proper foundation for the study of advanced arithmetic. Under this condition it will be necessary to eliminate some topics in the eighth grade algebra.

If arithmetic is to be given in both the seventh and eighth years it will be possible to insure greater efficiency along industrial and commercial lines through reviewing, drilling on, emphasizing, and enlarging upon those processes and applications which are considered the essentials. Without attempting to follow the syllabus it is possible to introduce some algebra in the solution of problems in mensuration and interest by means of the simple equation and the use of the unknown quantity x.

Give plenty of oral drill in getting approximate results. This will tend to reduce error in computation. In business it is customary to apply some kind of a check to every result obtained. No good mechanic or business man would think of letting his results stand without some checking. Pupils should acquire the verification habit.

Mental arithmetic should occupy a large share of the time. Never allow pupils to use pencil if, in your judgment, the result should be obtained mentally.

In general, papers should be marked, as they are in business, largely by the accuracy of the result. If the result is wrong the paper is wrong. If the problem requires some interpretation a teacher may quite properly mark both for accuracy and for method.

Teachers should endeavor to get outside of the book and to have a large amount of drill material ready for each exercise. Use the mimeograph, or some other duplicating machine in the preparation of lessons. Teachers should, if possible, learn the business methods used by a mason, carpenter, paper hanger, painter, architect, banker, etc. Original problems can be obtained from these men and pupils may be sent to them to verify their own methods. The problems should be real if they pretend to be so. They should relate when possible, to the child's daily environment and to his other school work. They should be concerned with the school shop, drawing, and laboratory work. They should touch the questions of the supply of food, clothing, and shelter as related to our farms, shops, transportation facilities, to typical industries, and distributing centers.

Visits to the shops of craftsmen and trips to factories, banks, and business houses will clear up in the minds of the pupils the technical points involved in such problems and will add new interest to the work. The daily activities of our people must be drawn upon to make the arithmetic interesting, informational, and practical.

FIRST HALF.

1. Continue application of percentage—
   (a) Interest, including commercial paper: (1) Six per cent method and short methods based upon it; (2) bank discount; (3) compound interest as applied to savings-bank accounts. Use tables in compound interest. The operations of depositing money, drawing checks, and figuring interest should become as real as the school can make them.
   (b) Problems should be many but simple. Many should be oral. In many cases discuss the solution without requiring any actual computation. Drill on giving approximate results.
2. Ratio and simple proportion.

Ratio has some use apart from being an introduction to proportion. Fertilizers, metals, and liquids are mixed in given ratios. Proportion may be merely one method of writing simple equation. The relation of numbers expressed as ratios should be shown and simple problems involving proportion should be given. The use of the equation and the unknown quantity x should be used.

3. Powers and roots—
   (a) Memorize squares of numbers to 25. Oral drill in their use.
   (b) Square root and its applications in determining dimensions from areas of squares. Also its use as applied to the right triangle. Square root is necessary for the purposes of mensuration. However, its study has more value in training for reasoning than in its practical use, since many who use it employ tables. Pupils can comprehend it by the use of the diagram or by the formula.
4. Many miscellaneous and review problems should be given. The use of the equation and the unknown quantity should be used, whenever convenient, in their solution.

SECOND HALF.

1. Review the following tables until the pupils use them readily in their work. In the review of the tables of measures some attention might be given to the origin of such measures as yard, foot, inch, quart, acre, etc. In this way the pupil will find the work taking on new interest. (1) Measure of length, (a) linear; (2) measure of area, (a) square; (3) measure of contents and capacity, (a) cubic, (b) dry, and (c) liquid; (4) measure of weight, (a) avoirdupois; (5) measure of quantity, (a) paper; (6) other tables, (a) circular, (b) time; (7) money, (a) United States, (b) English.
2. Review of, and drill in mensuration, using practical problems involving surfaces of (1) parallelogram, (2) rectangle, (3) triangle, (4) circle, (5) trapezoid and contents of such solids as (a) cube, (b) sphere, (c) cylinder, (d) pyramid, (e) cone.

Strict geometrical demonstrations can not be given, but enough objective work can be offered to make the matter clear.

3. Begin by using problems all of which require the use of the same table and then drill on mixed problems, making them practical and real as far as possible. Teachers are advised that it is of little value for the pupil to memorize tables that are not to be used in practical life, or to commit to memory numerous equivalents that are readily found in the textbook when needed.

EIGHTH YEAR.

FIRST HALF.

1. Rapid calculation work.

In this drill include daily speed exercises. Set a reasonable time limit and hold the class up to it. This will fit the pupils for pressure work which is bound to come in business life. It may include practice in the following:

(a) Addition until pupils can add at the rate of from 75 to 100 figures per minute. Use group method.

Horizontal addition should be practiced. Example: 10 pieces of cloth—38, 82, 91, 46, 53, 67, 84, 75, 65, 69=670, at 50c=$335.

(b) Multiplication, when multiplier is 11, 22, 33, 44, etc. Example: 892×11=2 as unit figure; (9+2=11) 1 as tens; (8+9+1 carried=8) 8 as hundreds; (8+1 carried=9) 9 as thousands; final result 9812. Example: 596×22=13112. Same as preceding process except that each addition is multiplied by two before carried figure is added.[1]

(c) Cost of articles sold by ton of 2000 pounds. Example: 8942 pounds at $25.50 per ton=$8.942×(½ of $25.50)=$114.01.

(d) Interest practice, short method given above. Only a few practical short methods should be given. This work will help to sustain interest and develop rapid thinking power of concentrated effort.

2. Business forms.

Some drills should be given in making such forms as the following: (a) invoices or bills; (b) statements; (c) account sales—in commission.

It is not expected that the pupil will be taught an elaborate form of bookkeeping; but everyone should know how to keep simple accounts. These can easily relate to the income and expenditures of daily life in the home and on the farm.

SECOND HALF.

Stocks and bonds:

Pupils should know what a corporation is, its chief officials, its organizaton, its stocks or bonds, and its methods of declaring and paying dividends.

(a) Cost of a given number of shares at a given market value.

(b) Number of shares or bonds that can be bought for a fixed sum.

(c) Rate of income on an investment in stocks or bonds.

Insurance:

Insurance, fire, life, and accident, is an important factor in our life, and yet the technicalities in various types of policies, especially those of life insurance, are so difficult of comprehension that the school can hope to do no more than to give a general conception of the work of the various kinds of companies and to confine the problems to the simplest practical cases that the people need to know about.

Taxation:

The subject of taxation has a close relationship to civics, and when combined with it has a real interest to the pupils. It should be treated from the standpoint of local conditions.

Algebraic notation:

Algebra has some place in this grade, especially for the boys in the industrial course. Simple formulas are now quite common in the various mechanical trade papers. Many of these boys will have use for various mechanics' pocket tables, which involve the elementary operations and linear equations. Em-

[1] The committee does not feel called upon to comment upon such features as this. It must say, however, that the carrying of this work beyond 11 is not warranted.

phasis should be laid upon the study of formulas, so that the simple algebraic expressions of the trade journals and mechanics' handbooks may be read easily.

RELATION TO MANUAL ARTS, HOUSEHOLD ARTS, AND AGRICULTURE.

Those schools that offer instruction in household and manual arts and agriculture have an excellent opportunity to relate this instruction to the book-work. A series of problems vitally relating to cooking, sewing, woodworking, mechanical drawing, and farming may be worked out. Such problems will not only touch the school life of the pupils but also the home life.

The suggestive outlines showing this relationship are not limited to a single year; neither are they necessarily entirely taught by the teacher of arithmetic. They furnish a splendid field for coöperative effort between the special teachers and the room teachers. Each might well know what the other is doing, and attempt to fix in the pupil's mind the applications of arithmetic to school and home life.

Portions of these outlines are common to the needs of both sexes. Certainly both should have a knowledge of home accounting. However, the division of hand activities into home-making courses for girls and manual training and agricultural courses for boys means that the most effective correlation will be brought about by a differentiation of the arithmetic according to sex and occupational interests.

OUTLINE RELATING TO HOUSEHOLD ARTS.

1. Simple domestic bookkeeping, including daily, monthly, and yearly supplies for the kitchen.
2. Calculation of the prices of foods bought in large or in small quantities as related to the cost of meals for the home.
3. Study of the simple measurements as needed in the household, including measurements of materials used in food and clothing.
4. Cost of furnishing a kitchen, laundry, dining room, living room, and bedroom, with the expense of renovating and maintaining them.
5. Relative cost of different systems of heating and lighting; reading a gas meter.
6. General questions of maintenance of the home, including rent, water, taxes, insurance, and interest on the mortgage.

OUTLINE RELATING TO MANUAL ARTS.

1. Adding the detailed dimensions on the school sketches and drawings for the "over-all" dimensions.
2. Figuring the cost of the materials used in the article made, including the lumber, screws, stain, and varnish.
3. Working out a more elaborate cost slip of the object made; i. e., reckoning the compensation for labor, the cost of power, the interest on the money invested in the school shop, etc., as well as the bill for materials.
4. Making out a bill for the general lumber supply of the school, using the actual market prices.
5. Laying out of simple geometrical forms in wood; i. e., hexagonal taboret—calling attention to the value in degrees of the angles.
6. Proportion as applied to the speed of the wood-turning lathe, taking the speed of the motor as a basis.

OUTLINE RELATING TO AGRICULTURE.

1. Actual measurements by the class of small plots of land, calculating their area. Measurements of walls, fences, and calculating their cost.

2. Measurements of and calculating the capacity of tanks, cisterns, watering troughs, silos, storage bins, and barrels. Data may be obtained from the home equipment.

3. Special computation suited to farm practice such as income from dairy cows, feed rations, farm labor.

4. Business forms, such as inventories, time books, stock accounts, cash accounts, notes, receipts.

5. Application of percentage in study of soil composition, ratio and proportion in relation to animal feeds, practice in estimating amount of material for such spraying; i. e., emulsions, poisons, and fungicides.

6. List of materials showing cost of such farm buildings as house, barn, poultry house, corncrib.

A CITY COURSE OF STUDY.[1]

GRADE 1.

SECOND HALF YEAR.

Numbers through 10: counting forward and backward; relative values of numbers which make 10; writing the names of numbers in words and figures.

GRADE 2.

FIRST HALF YEAR.

All the work of this grade should be objective.

Counting by tens, fives and ones to 100; relative values of numbers within 100.

Numbers through 20: counting by twos, fours and fives. Counting backward by ones, ½, ¼, and ⅓ of numbers from 1 to 20, inclusive, which give an integer as result.

Measures used—inch and foot.

SECOND HALF YEAR.

Addition and subtraction of numbers through 14; multiplication and division through 20; writing numbers through 100; ½, ¼, and ⅓ of numbers from 1 to 20, giving an integer as result. Addition of single columns, using the combinations learned.

Measures used: Inch, foot, square inch, square foot, pint, quart.

Review very thoroughly the work of the first half year. Teach addition and subtraction of numbers through 14: multiplication and division through 20. Teach signs for all processes, using the terms *and, less, times, contains,* and *equals.*

Simple concrete problems should be given, using the combinations learned. The order of presentation should be: 1. Objects, in learning combinations. 2. Concrete examples (oral) with objects present. 3. Representation of combinations by figures. 4. Recalling combinations without objects; thorough memorizing of results. 5. Concrete problems without the use of objects. These problems should be very simple and based upon the child's experience. Very little written work should be required and the result should be stated in a sentence without any attempt to show the process. Readiness in finding the

[1] Prepared for the schools of Indianapolis, Ind. The course as here given is necessarily much abridged.

result of the combination of any two numbers is to be considered of first importance in this grade, because this knowledge must be at command in all succeeding stages of advancement. The addition of short columns, using only the combinations learned, gives additional practice; it is an excellent method for review.

GRADE 3.

FIRST HALF YEAR.

Addition and substraction of numbers through twenty; multiplication and division tables through fours; abstract addition, two columns; writing of numbers through one thousand; Roman notation through one hundred. Fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$.

Measures used: Inch, foot, square inch, pint, quart, gallon.

SECOND HALF YEAR.

Addition and subtraction by "endings" through 2+9. The four fundamental processes—multiplication and division tables through fives. Multiplication and division of abstract numbers through thousands; use 2, 3, 4, and 5 as divisors. Addition and subtraction of United States money. Writing numbers through ten thousand. Roman notation through one hundred.

Application of fundamental processes to simple concrete problems of one step.

Measures used: Inch, foot, yard, square inch; pint, quart, gallon; peck, bushel; second, minute, hour, day, week, month, year. Use actual measures. In connection with the units of time, call attention to the method of telling the time of day by the clock.

GRADE 4.

FIRST HALF YEAR.

Addition and subtraction by "endings" through 4+9. Multiplication and division tables through nines. Multiplication of abstract numbers by two digits. Short division, using for divisors all numbers below 10. Addition, subtraction, and multiplication of United States money. Writing numbers through millions.

Fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. Aliquot parts, $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{10}$, as applied to 100 only.

Measures as in Grade 3, and in addition, the ounce and pound. Use actual measures.

SECOND HALF YEAR.

Addition and subtraction by "endings" through 9+9. Multiplication by three figures. Long division. United States money, addition, subtraction, multiplication, and division.

Fractions, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{6}$.

Pupils of this grade should become accurate and skillful in addition and subtraction of integers. Begin the work of the term by making sure that pupils can add and subtract by endings unfailingly. Subtraction by endings is especially likely to be neglected. Require pupils always to check their addition by adding the columns in reverse order. Similarly subtraction should be checked by the reverse process of addition. Thus $13 - 9 = 4$, is checked by the process $9 + 4 = 13$. Do not allow pupils to indicate "borrowing" by changing the digits in the minuend.

At the beginning of the term, make sure that every pupil has mastered the multiplication combinations through the tables of twelve—not only when given in serial order (9×3, 9×4, 9×5, etc.), but when selected at random (9×4, 7×9, 12×11, etc.).

Use multipliers of not more than three figures in this grade. Introduce zero into the multiplier often, as it is troublesome.

Pupils should always check multiplication by going over the work carefully, adding the partial products in the order reverse to the first addition.

It should be remembered that pupils will fail in the long division process as long as they have not mastered multiplication and subtraction.

Pupils should always check long division by going over their work carefully, using the reverse process of addition for each subtraction.

To insure mastery of the division process, after the process is acquired give a problem each day, either in long or short division, during the remainder of the term.

Mental or oral arithmetic should be given daily in abstract and concrete problems, for at least one-third of the arithmetic period.

GRADE 5.

FIRST HALF YEAR:

Review of essential processes taught in preceding grades. Long division. Reduction, addition, subtraction, and division of fractions. Aliquot parts: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{8}$. Mental arithmetic.

Pupils should be able automatically to add and subtract by endings and to use the multiplication combinations through 12 times 12, before beginning the fractional work. While the fractional work is being developed in class time, the written work should consist almost entirely of review problems. Enough practice should be given in long division to enable pupils to use the process accurately and fairly rapidly. Insist upon the use of short division where the divisor is less than 13.

The chief difficulty that pupils have in acquiring the fractional processes is to interpret clearly the unfamiliar, and so perplexing forms and terms used. The problem of the teacher therefore is to enable pupils to interpret these conventional symbols into terms of their own experience. To this end pupils should first learn to objectify the common fractional quantities, halves, thirds, fourths, sixths, eighths, and twelfths. Pupils should compare these fractional parts when applied to actual objects (oranges, apples, sheets of paper, etc.), and to representations of objects by drawings. Drawings should be used very largely for objective development in this grade, as they make possible a much wider application of the fractional parts than would be possible with a few objects. The drawing, moreover, is the natural step between the object and the abstract symbol. Whenever a drawing is used, the child should think of it as representing some definite object.

Mixed numbers, proper and improper fractions.—Develop these terms objectively by paper folding or from drawings. Pupils at first read a mixed number, $2\frac{2}{3}$, as 2 whole and $\frac{2}{3}$ of a whole. They should realize that an improper fraction is equivalent to a mixed number with the wholes cut into the same sized parts as those indicated by the fraction and all of these parts combined.

Reduction.—Pupils should now specialize in the following processes until they are skillful in the use of them, approaching each from the objective point of view.

a. Reduction of integers and mixed numbers to fractions.

b. Reduction of improper fractions to integers or mixed numbers.

c. Reduction of fractions to higher and lower terms. The work in this last topic should be especially thorough, as addition, subtraction, and division of fractions usually involve one or both processes.

Addition, subtraction, and division of fractions.—Pupils that have had the above preparation will have no difficulty in acquiring skill in the use of these processes if abundant practice be given. It should be recognized that the business world seldom deals with fractions whose denominators are higher than 16, preferring to use decimals for all such fractions. This should cause the teacher to drill largely upon fractions of the larger denominations (with small denominators).

Finding the least common denominator (by inspection only) should be taught. Denominators too large for this method should not be used.

SECOND HALF YEAR.

Review of preceding essential processes. Multiplication of fractions. Addition, subtraction, and division of decimals. Aliquot parts: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{1}{12}$, $\frac{1}{16}$. Mental arithmetic.

Make clear to pupils that a fraction may be expressed in a variety of ways: (1) Five hundredths; (2) 5 hundredths; (3) $\frac{5}{100}$; (4) .05.

The last way is called the decimal form, because it is used only to express fractions whose denominator is ten or a multiple of ten.

At this point lead pupils to see that a fraction has two meanings:

First. The meaning that they, as primary school children, were taught to attach to it—namely, that a fraction expresses a certain number of equal parts of a unit.

Second. The meaning that the mathematician often attaches to it—namely, that a fraction is an expression of division, in which the numerator is the dividend and the denominator is the divisor.

GRADE 6.

FIRST HALF YEAR.

Review of essentials. Multiplication of decimals. Percentage, first case. Denominate numbers and measurements. Bills and receipts. Mental arithmetic.

Teachers should, at the outset, determine which pupils can not add and subtract readily and use the multiplication combinations automatically, and assist them to overcome these fundamental weaknesses. The first two or three weeks should be used in a thorough review of the essential processes. Teachers should, throughout the term, emphasize the work in all the decimal processes so that the pupils may acquire skill in the use of them before leaving the grade.

In this grade give only the first view of percentage. For this reason confine the work to an explanation of the way of reading and writing per cents, to teaching the connection between per cents and integers, and to finding given per cents of numbers. Keep the work simple, using only whole numbers of per cents. For example, use 4%, 5%, 6%, etc., but not $4\frac{1}{2}\%$, $6\frac{1}{4}\%$, etc., in this grade. Some suggestions:

(*a*) Teach percentage as a phase of multiplication of decimals.

(*b*) Teach the idea of per cent by finding, orally and in writing, hundredths of given numbers, expressing the written work at first by the common and then by the decimal form of the fraction. After the pupils

have a clear idea of finding hundredths of a number decimally, give them the percentage form of notation and expression; i. e., tell them that 0.05 is written 5%, etc. They should learn to associate hundredths and per cents.

(c) Give the pupils wide experience in changing decimals to per cents and per cents to decimals, using integral numbers of hundredths.

(d) Give the pupils wide experience in finding by the decimal process a given per cent of a given number. Do not introduce during this term the difficulty of deciding whether or not to use the common fraction or the decimal fraction equivalent of a given per cent; that is, use always the decimal fractions.

Denominate numbers and measurements should receive a thorough treatment in a practical way in this grade. Teach the necessary tables of denominate numbers. Some of these tables the pupils already know, whereas others they will now meet for the first time.

(a) The pupils should memorize thoroughly these tables, excepting that every item which modern business does not require the average man to know should be excluded; more definitely teach the following tables: time, weight, dry measure, liquid measure, linear measure, square measure, and cubic measure.

Ask the pupils to bring to school bills that have been used in business transactions. Talk them over with the class, showing the advantage of the conventional forms of ruling, heading, receipting, etc. The pupils should be asked to make out bills first on blank forms, if they can be obtained, afterwards on paper which they have ruled themselves. Let their imagination be used to make these business transactions represent actual transactions as far as possible. Let it be imagined or represented that the teacher or one of the members of the class has sold the rest of the class a bill of goods or done a piece of work for them, or rented a house to them. The resulting bill should contain the real names of the parties to the transactions. The pupils should learn how to receipt a bill properly and also how to give a receipt for any obligation; that is, an account settled, rent paid, wages received, and part-payment receipt.

All processes and subjects taught in this grade should be introduced by simple work given orally. After a principle has been taught a large number of simple exercises should be given in mental arithmetic, so that the pupils can become sure and skillful in the application of the process. Mental arithmetic should be given daily for one-third of the recitation period.

SECOND HALF YEAR.

Review of all essential processes previously taught, as follows:

(a) Addition and multiplication combinations.

(b) Notation and numeration of integers, Arabic and Roman; fractions, common and decimal.

(c) Reduction. Changing denominate numbers to higher and lower denominations; a common and decimal fraction to higher and lower terms; changing mixed numbers to improper fractions, and the reverse process; changing decimals into common fractions and common fractions into decimals.

(d) Addition, subtraction, multiplication, and division of integers; common and decimal fractions; denominate numbers.

(e) The use of the common aliquot parts.

Percentage.

Measurements.

Simple bills, accounts, and receipts.

Mental arithmetic.

The purpose of the review work in this grade is to establish a strong foundation for future mathematical study by making pupils sure and skillful in the use of the essential processes previously studied, and by revealing to them the unity of the subject as based upon the two general underlying principles.

At the outset the teacher should make sure that every pupil can add and subtract accurately and use the multiplication combinations automatically. All cases of "arrested development"—such as counting in adding, adding instead of using the multiplication combinations, and writing down numbers to be added to the next higher order in multiplication—should be earnestly sought out and corrected. The teacher, particularly of this grade, should feel that unless this fundamental work is thoroughly accomplished her work in arithmetic is, to a considerable extent, a failure.

Pupils should now be able to read and write the quantities, appearing under various forms, which they will need to use. They will probably have no difficulty with common fractions and denominate numbers, but they may need considerable drill in integers, Arabic and Roman, and in decimals. Their knowledge of the Roman system will be used almost exclusively in connection with the numbering of chapters in books, and it should be taught with this in view. Drill only upon numbers below C.

In the decimal notation considerable drill may be necessary before the pupils will readily associate the name of the order with the number of decimal places.

Sufficient drill should be given to enable pupils to reduce denominate numbers to higher and lower denominations and to give them proficiency in changing fractions to other fractional forms; common and decimal fractions to higher and lower terms; improper fractions to mixed numbers, etc.

It should be the aim of the teacher to keep the percentage work entirely on the thought side, avoiding all formulas or rules. The work includes the relation between common fractions, decimals, and per cents; the finding of a given per cent of a number; the finding of a number when a part is given; and the finding what part one number is of another.

"Gain and loss" should not be taught as an independent topic, for the problems which come under this head can be classified with the three kinds of percentage problems mentioned above. If it is clearly established with the child, by means of blocks or by representation by drawings, that the selling price is always made up of the cost with a part of itself added or taken away, according as there has been a gain or a loss, he will have no difficulty with these problems.

In connection with the percentage work, pupils should acquire facility in the use of the common aliquot parts. Those that are easily computed (as $\frac{3}{5}=60\%$) should not be committed to memory, neither should those that are not commonly used (as $\frac{5}{16}=31\frac{1}{4}\%$). Pupils should learn, however, to compute quickly the decimal equivalent of any fraction, by expressing the numerator as a decimal and dividing by the denominator. An excellent drill consists in counting by fractional parts, giving at the same time the decimal or per cent equivalents.

In measurements, review the work with the rectangle, square, triangle, cube, distinguishing them from each other and associating these geometrical names with the figure already familiar.

The problems under this head should be practical. Require pupils frequently to make their own measurements in order to obtain the conditions of the problems they are to solve. Illustration: Find the dimensions of your schoolroom. What is the total area of the walls and ceiling? How much window surface? How much blackboard surface? Determine the cost of sodding the yard of your school and inclosing the school yard with a fence.

One good way of treating this topic is to work out problems in connection with building and equipping a house or the school building. If a building is being erected in the neighborhood, it should be used for this work in order that the problems may be real.

In order to train pupils to keep simple accounts—an accomplishment that everyone should possess—teachers are requested to have at least one exercise a week in which the children make a record of their income and expenses. This exercise need not occupy an entire period. Except when class instruction is needed, the records may be made before school, or at odd times.

The accounts should be of actual moneys belonging to the pupils, if that is possible. It is better to have a briefer account and have it a real one than to fill it with fanciful transactions or family transactions. The accounts should indicate briefly, but adequately, the sources and dispositions of income; not Rec. from Aunt Mary, but Gift from Aunt Mary; not Paid out, but Paid for candy; not Earned, but Shoveling snow, etc. The headings at the top of the pages should be uniform through each book—either Received—Paid, or Income—Outgo, or Receipts—Payments. Balances should be computed at the close of each month. These balances should be properly brought down by use of red lines, if possible, and carried forward to the next month's account. Begin each month on two new pages, with proper statements of amounts brought forward from previous month. Have a uniform use of capitals and punctuation. Pupils may keep memorandum slips, but entries in the account books should be made in school, with ink, and accounts should not be copied. Accounts through vacations should be kept in memorandum form and entered in the books in school after vacation. The accounts will be opened October 1 and February 1 and be continued through the 6A grade only, including vacations. Account books will be furnished this grade.

Ask the pupils to bring to school bills that have been used in business transactions. This work is review, and pupils should become proficient in making and receipting bills of various sorts that are used in the common business of life. Make the work practical, and frequently during the term give practice in this work.

All processes and subjects taught in this grade should be introduced by simple work given orally. After a principle has been taught a large number of simple problems should be given in mental arithmetic exercises, so that the pupils may become sure and skillful in the application of the principle. Mental arithmetic should be given daily for about one-third of the recitation period.

Grade 7.

FIRST HALF YEAR.

Review of essential processes. Applications of percentage (commercial discount, commission, taxes, simple interest). Constructional geometry. Mental arithmetic.

First make sure that all pupils can use the fundamental processes skillfully. If weaknesses are revealed, test for addition endings and multiplication combinations and follow up all deficiencies until they are overcome.

Devote the first two or three weeks—more or less, according to the class—to a review of the essential processes. It is only when a good foundation is established in this review work that the best results in the advance work can be assured.

Do not allow pupils to become confused by the new terminology found in the applications of percentage to various kinds of business. Cause them to see that

these involve the identical principles and processes which they used in their study of percentage in the preceding grades. Keep the work simple, practical, and in accordance with present business methods. Train pupils to investigate by polite and pointed inquiries the different kinds of business studied. The information thus gained should be reported to the class in an interesting and profitable manner.

The subject of commercial discount is of great value because of its extensive use " from wholesale transactions down to bargain sales." Pupils should understand some of the reasons for allowing discounts. Buying in large quantities; paying "cash down" or within a specified time, the usual deductions from list prices, etc. It would be well, for obvious reasons, to review bills and receipts in connection with the study of this topic.

Let the pupils interview commission merchants and obtain conditions for some practical problems, and, at the same time, a social interest in this important business. Eliminate all problems that state that the sum of money sent includes the commission and the price of the goods bought, as this is contrary to business practice.

The subject of taxes should be treated briefly. It should be shown that there is expense involved in conducting the affairs of a city. To make this concrete, use as an illustration the fact that the public schools are dependent upon the taxpayers for their support. Point out the necessity of each citizen paying his rightful share of the public expenses of government. The simple mathematical work in the subject may be developed by the analogy between levying a city tax and taxing the members of a ball team to meet its expenses. The attempt in the former case to apportion the taxes according to the individual's property, may be brought out by the contrast in this respect between these two illustrations of taxation.

Confine the work in this grade to the subject of municipal taxes, discussing the subject only in a simple way. Use the current tax rate in problems.

Present conditions so that the child can discover that interest is merely a form of percentage with the time element introduced.

One arithmetic period each week should be devoted to the geometry work. The purpose of the work is to teach pupils to use the few mechanical drawing instruments which they buy, and to acquaint them with some of the essential fundamental terms and principles of geometry.

SECOND HALF YEAR.

Review of essential processes.

Problems of the home. Grocery, meat market, department store problems. Making change. Cost of heating and lighting the home. Cost of furnishings for the home. Other problems relating to the cost of food, heat, light, clothing, etc., for the family.

Saving and investing money. Saving money. Investing money. Banking. Interest. Real estate. Loans.

Constructional geometry.

Mental arithmetic.

In the problems for the home this city provides data in a pamphlet on community arithmetic.

These problems are designed to give drill upon the essential processes through the use of such problems as are met by individuals in their daily life. They are to be solved upon the basis of current prices in every case. For this reason it is important that the teacher should get in advance a list of the items, together with the current prices. These problems are the kind all of us as

purchasers should be prepared to solve mentally, or if not so, then in writing with very brief notes; they are the kind that salesmen and clerks are called upon to solve in their daily work. The problems will be found useful for both mental drill and for written lessons.

The department-store problems are designed to meet the criticism of store men that both clerks and the public are unskillful in the solution of such simple problems as occur in the conduct of a department store.

Give problems showing that small daily and weekly savings will amount to large sums in time; that money may often be saved by making cash payments rather than paying by installments or buying on credit; that it is more economical to buy in reasonably large quantities; that the cheapest things are sometimes the most expensive in the long run.

Consider the various ways of investing money: Banks and trust companies; real estate; loaning money. Establish the economic principle that the higher the rate of interest offered by an investment, the greater the risk involved; also, that the element of safety in investments is of far greater importance than the amount of income from them.

The bank is the most commonly used means provided for the safe-keeping of money. Certain kinds of banks—for instance, savings banks and trust companies—not only guarantee to take care of all money left with them by depositors but they also pay a certain per cent of interest.

When practicable, pupils should go to a bank and ask some official to show them about the bank. Report what you learn to the class. Ascertain what officials there are in the bank, and the duties of each.

Many schools have found it interesting and profitable to organize school banks, electing the officers and carrying on a regular banking business, either with small amounts of real money placed on deposit by pupils and transferred by the teacher to some bank or trust company, or with imitation money, which can be made by cutting out rectangular pieces of paper for bills and circular pieces for coins, marking on each piece its denomination.

Teach the method of depositing money, of drawing checks, of borrowing money, and of discounting notes.

Not less than one arithmetic period a week should be devoted to the geometry work. The purpose of the work is to teach pupils to use the few mechanical drawing instruments which they buy and to acquaint them with some of the essential fundamental terms and principles of geometry.

The aim of each exercise should be clearly stated to the pupils and they should be led, so far as practicable, to think the successive steps to the conclusion. In other words, make it a thinking exercise rather than a mere dictation exercise. Encourage pupils to use their instruments as much as possible in the mensuration work in arithmetic, in the art work and manual training work, and whenever they will be serviceable.

GRADE 8.

FIRST HALF YEAR.

Review of essentials.

Problems relating to the industries of the city and State. Manufacturing interests of Indianapolis. Manufacturing interests of Indiana. Problems met in certain lines of work; some railroad problems; foundry problems; problems of buying and selling paper; making out a pay roll.

Paying and collecting money by checks, drafts, and money orders.

Generalized arithmetic and equations.

Proportion.

Mental arithmetic.[1]

Instead of the above, the Latin classes take algebra; however, in these classes also, drill in the previous arithmetic work so as to maintain efficiency.

In the eighth grade, problems relating to the interests of the city are prepared and are issued in pamphlet form.

SECOND HALF YEAR.

Review of essentials.

Generalized arithmetic and equations.

Stocks and bonds.

Insurance—fire, life and accident.

Cost and support of our Government. Cost of city schools. Cost of city government. Money raised by taxes. Money raised by sale of bonds. Money raised by special assessment.

In treating of stocks and bonds show that a great deal of the wealth of the country is invested in the various kinds of business which supply our wants and minister to our welfare. Name a number of the business enterprises in or near your community which are owned and controlled by single individuals; a number in which more than one person have invested their money. Show that much of the business of the country is now carried on by these corporations—for instance, railroads, trolley systems, and many of the large manufactories. The money contributed to carry on these industries is represented by bonds or certificates of stock, which are bought and sold like land or merchandise. Explain the fluctuation of stocks and the difference between stocks and bonds.

Pupils can find the market price of the more common stocks and bonds quoted in the daily newspapers, and while studying this subject they should follow these quotations from day to day. Explain the abbreviation "pfd." (preferred), and the difference between preferred and common stock. Most railroads and many other business enterprises issue these three forms of securities—bonds, preferred and common stock. Pupils should study these different forms of securities as issued by some local corporation, as a traction company or railroad.

The work in stocks and bonds will be limited to the finding of the cost of a given number of shares of stock or of a given number of bonds, at a given market price, and to the finding of the income from a given number of shares of stock or a given number of bonds.

Insurance should be studied for its informational rather than its mathematical value. Three kinds of insurance should be briefly and simply discussed—life, accident, and fire.

Pupils should understand the underlying principle in insurance; paying a relatively small, but carefully estimated sum of money to a company for their guarantee to pay a very much larger sum of money in case of a specified contingency.

They should understand the following terms: Policy, premium; "straight life" and "endowment" policies; "good" and "poor" risks.

The problems in the cost and support of the Government should be designed to maintain and increase efficiency in the essential processes of arithmetic through constant application and to convey information concerning matters of local or national interest, of which all citizens should have some knowledge. This aspect of the work is designed to correlate with the work of the civics class. The problems should afford opportunity for the use of the four fundamental operations, with integers, percentage, and some of its applications.

SUBCOMMITTEE 2. MATHEMATICS IN THE KINDERGARTEN.

In the early history of the kindergarten as an educational movement, there was a somewhat unanimous opinion current regarding the direct mathematical training and the instructions to be given to the children through concrete experiences with Frœbel's educational materials.

The gifts and occupations (the technical names given to Frœbel's play materials) form a related series planned to meet the creative and constructive impulses of the child, through activities involving the analysis and synthesis of geometric forms. The gifts begin with the ball (sphere), the cube, and the cylinder, progressing through the analysis of these to the study of surfaces, lines, and points. The occupations are based upon such activities as perforating, sewing, drawing, weaving, the folding and cutting of paper, and modeling. They reverse the geometric evolution, embodying the synthesis of form from the point, through lines and surfaces, back to the solid.

The traditional method of using these materials in the early history of the kindergarten tended toward a much more direct process of instruction, bringing to the children a consciousness of the geometric relations embodied in the gifts and occupations.

As the modern primary school reduced the amount of conscious arithmetical knowledge and instruction in the course of study for the younger children, a parallel movement took place in the kindergarten. As a consequence, the more direct method of mathematical instruction, which the geometric basis of the Frœbelian materials made possible, gradually gave place to a more indirect approach to number, measurements, and geometric relations, through a more incidental, organic, and fundamental emphasis upon these aspects of the materials in the moral activities of work and play.

The present tendency seems to point toward a valuation of the mathematical possibilities of the Frœbelian materials as means to a higher end; that is, as structural, functional, or organic means necessary to realize the playful, the creative, or constructive impulses of the child through which these mathematical values may begin to come to consciousness.

The fact that the Frœbelian gifts and occupations are based upon a geometric analysis and synthesis of form, need not reduce or interfere with their creative and playful opportunities, except in the hands of a kindergartner who clings to the formal use of them as a means of direct mathematical instruction. On the other hand, the very fact that their structure and use involve such unconscious activities as counting, addition, subtraction, multiplication, and division, some consciousness of these mental processes and their results will grad-

ually come to the child whether the teacher emphasizes or ignores them.

The very nature of the material familiarizes the child, through playful concrete experiences, with the mental processes involved in the solution of problems in fractions and all the elementary activities of mathematics which may be brought to consciousness later through the more direct methods of instruction in the elementary school. To what degree the mathematical values involved in these playful activities should be brought to consciousness at the kindergarten period, is still under discussion, with much less decided differences of opinion than would have been apparent five or ten years ago.

While the structural and functional use of the activities of counting, measuring, adding, subtracting, and dividing seems to be on the increase, there is an equally evident growth in the appreciating of the fact that they must not be left to chance experience without due attention to ways and means for providing opportunities for further progress. In other words, there is a decided feeling that in mathematical experiences, as with all others in the kindergarten, the teacher must provide conditions which insure steady progress from the simple to the more complex, from the unconscious activities involving mathematical values to the more conscious abstraction of these, and to a continuous growth in the appreciation and control of the mathematical possibilities which are normal to the child at this period.

As a knowledge of mathematics in some form underlies all industrial activities of mankind and in the foundation of all true proportion in the art world, the kindergarten brings the child to a gradual consciousness of numerical and geometric relations, the former by such features as simple counting, measuring, and adding, as his work and play may demand, and the latter by the use of the geometric gifts and not by abstract exercises. In this way the younger children get arithmetical and geometric impressions incidentally, and the older children come into a more definite knowledge of mathematical relations, and thus begin an intelligent mastery of the material world.

The kindergarten which represents the progressive school makes no attempt to teach mathematics in a formal or direct way, since the child of kindergarten age is not interested in and can not grasp anything so abstract as number or geometric form apart from the use he may need to make of each in carrying on his play activities successfully. But in his experimenting and play with different materials he discovers certain facts in regard to number and special relations; for example, that if he uses four of his eight building blocks to make a square table, he has four left to use as chairs. Later he will make use of the knowledge thus gained, and will acquire further knowledge of the same sort in his efforts to carry out his ideas and purposes.

Often the hand work requires the division of material into halves, quarters, etc., or some material needs to be measured and cut to fit. Through such experiences the child acquires a working knowledge of certain satisfactory methods of dividing and measuring material. Furthermore, he becomes familiar with a number of geometric solids through using them in building, and he learns to select the kind that will best express his idea. The same is true of his work with paper cut in the form of squares, circles, and triangles. The games sometimes call for the grouping of children in threes or fours, and there are many occasions when counting is advantageous. The child in the kindergarten thus gains considerable knowledge which may be termed mathematical. Such knowledge comes at first incidentally and unconsciously through play, and later as the result of conscious efforts to reach ends which appeal to him as valuable.

After due investigation an effort was made by this committee to arrive at some conclusion regarding the amount of mathematical knowledge which kindergarten children could acquire, without direct instruction, through the normal activities of work and play. As a result the committee has agreed upon the following:

(1) Ability to count up to 35 or 40 can be secured through the children's helping to keep the daily record of attendance.

(2) Ability to know and name correctly the sphere, cube, and cylinder, and the most characteristic surface forms such as circles, squares, rectangles, and the right triangles.

(3) Ability to know groups of objects up to five or six.

(4) Ability to know, construct, and use intelligently halves, thirds, and quarters, by the help of blocks and the kindergarten occupations.

(5) Ability to add, subtract, divide, and multiply small numbers through constructive play. For example: (*a*) 2+2, 2+3, 2+4, 3+3, 3+4, 4+4, 5+5; (*b*) 2×2, 2×3, 2×4. (*c*) 4−2, 6−3, 8−4.

As to method, the committee calls attention to the following points:

(*a*) The concreteness of the work.

(*b*) The work is functional and structural, as means to an end, through the activities of work and play.

(*c*) Knowledge is acquired through self-active experience.

(*d*) In an incidental but not accidental fashion, as a result of definitely planning the experiences which involve steady progress in the use and control of mathematical facts, the child comes to realize ends that are of worth and interest.

That a better knowledge of the child is leading to a greater unanimity of opinion is quite evident from this report, since it is prepared by members who were selected because they were supposed to represent widely different points of view that obtained a few years since.

SUBCOMMITTEE 3. MATHEMATICS IN GRADES ONE TO SIX.

I. INTRODUCTION.

As four definite topics were assigned the committee, it was agreed to divide them among the four members other than the chairman, so that each could do intensive work on one topic. The chairman was thus left the task of checking each by doing extensive work in all the fields designated. The topics were consequently assigned as follows:

Topic a.—The organization of schools and the general relation of each kind of school to the others.—Miss Julia Martin, Howard University, Washington, D. C.

Topic b.—The mathematical curriculum in each type of school.—Miss Harriet Peet, State Normal School, Salem, Mass.

Topic c.—The question of examinations from the point of view of the school.—Dr. C. W. Stone, State Normal School, Farmville, Va.

Topic d.—The methods employed in teaching mathematics.—Prof. Henry Suzzallo, Teachers College, Columbia University, New York City.

The reports on these topics naturally overlap in certain investigations. As originally prepared they contained numerous graphs and statistical tables, embodying many scientific data, and only the exigencies of publication prevent their appearing in complete form. All have necessarily been condensed for present publication.

The chairman gathered her data in narrower fields, but in the same way as did her colleagues, namely, by—

(1) Personal conferences during eight months with hundreds of teachers, met at institutes and in conventions, at teachers' meetings and study clubs. (2) Correspondence with teacher friends in almost every section of the United States. (3) Study of text books and manuals in mathematics, published during every decade since 1800. (4) Study of methods in mathematics as given in all forms of pedagogical writings during the last 50 years. (5) Examination of courses of study issued by four State departments, five State normal schools, six city systems, and five private schools. (6) Testing several fifth grades in one city by means of Dr. Stone's "standardized tests." (7) A careful study of examination questions issued by teachers, superintendents, and State departments. (8) A questionnaire given to about 300 normal-school students.

The questionnaire was formulated to secure evidence on topics *b*, *c*, and *d*. Plainly, during the last 10 to 15 years (when the students questioned were in grades one to six) arithmetic grew yearly a more vital part of the course of study, being "the subject most emphasized," being most often "the subject governing promotion," and being the subject in which the majority of the so-called "brightest" pupils "excelled."

The course of study as recalled limits the fundamental operations applied to both integers and fractions, leaving all "extras" to the seventh and eighth grades. One interesting fact was the great uni-

formity in distribution of arithmetic topics; the students questioned had had their early training in at least 10 different States, and in no less than 100 different schools.

On the question of examination over 87 per cent had to take examinations, the majority taking them either monthly or semi-annually. In general the examination counted for one-third to one-half of a pupil's promotion average. Eighty-two per cent objected to examinations, usually because the student was "nervous," "worried," "afraid," or filled with "dread," but occasionally because the test seemed unfair or caused cheating.

These same students imagine (in a few cases only they "know") that only 78 per cent of the teachers favored examinations, fully one-third of this number favoring them only because "it was a duty to be met." At least 22 per cent of teachers are thought to object to examinations.

The questionnaire gave little help on the question of change in methods, since students were plainly influenced by very recent work, or else could not write out the explanations they had once had. Ratio work was evidently little understood, students almost invariably using one as the only basis of comparison. If they had ratio work since they left the intermediate grades, it must have appeared so difficult that they think young pupils should still be taught the to them easier or simpler way of returning to one each time.

The chairman made her report in full before the individual reports were received (or, at any rate, opened), so that the unanimity of opinion as to present conditions and marked tendencies in the teaching of mathematics displayed in the five separate reports prepared at great distance from one another, based upon different sources of information, and frequently overlapping, gives one confidence in the validity of the results indicated.

In recasting the whole for publication, extra effort was given to eliminating whatever did not bear directly upon the topic of mathematics as taught in grades one to six of the general elementary schools, public and private.

TOPIC A: THE ORGANIZATION OF SCHOOLS AND THE GENERAL RELATION OF EACH KIND OF SCHOOL TO THE OTHERS.

A fair degree of uniformity marks the organization of schools in the different States of the Union, only such variations being noticeable as would naturally come from difference in (*a*) age of the settled territory, (*b*) density of population, and (*c*) inherited ideals.

First, we note two large classes of schools, public and private. The first group naturally divides again into three subdivisions, namely—

(*a*) Those found in rural communities, and called "ungraded schools," "country schools," "rural schools," "district schools," or

"parish schools," according to the locality and the educational system. As rural communities increase their wealth, their ambition for educational advantages, and their interest in educational problems, new forms of these rural schools come into existence, called "semi-graded schools," "consolidated district schools," "township high schools," etc.

(*b*) So-called "graded schools," found in smaller as well as in large cities. Such a city system usually consists of an "elementary school"—eight years of work, divided into primary, including kindergarten if given, intermediate, and grammar grades—and a "secondary school" or "high school."

(*c*) Those schools which the State or Government at large supports, the only ones of these which include grades one to six being (1) schools for abnormal and indigent children and (2) elementary schools in connection with normal schools and with such State universities as have a department of education and maintain a "practice school."

The relationship between these subclasses of public schools is very slight, rural schools being largely autocratic, in spite of existing school boards and county superintendents, whereas graded systems, also (nominally) under the jurisdiction of the county superintendent, are in smaller cities much more democratic, though perhaps plutocratic in large cities where specialists supervise the work, while schools of the third class might, in a similar way, be denominated as bureaucratic. Of course, schools for defectives, incorrigibles, etc., do not pretend to employ the same materials of education nor the same standards of scholarship as do the other classes of schools. Theoretically, if not actually, normal school and university "model schools" establish the standards for their State. They are organized to further a threefold function: (1) "Model" instruction for pupils attending, (2) opportunities for practice in teaching by the students of the main school, and (3) opportunities for testing out theories of education by professors of the different departments. Different schools may emphasize one or the other of these functions, but the three are still ever present. Because of the relationship between these functions, the curricula, the grading or organization, and the methods of instruction employed in a normal school, for example, are usually submitted to the faculty for discussion, so that the result represents a "cabinet" conclusion, and therefore is authoritative; usually, though not always, being adopted by other schools of the State.

Specialists who supervise the work in large cities have not the help of such discussions and must grow somewhat autocratic in the selection of subject matter and of method, the system being too large for them to get the expression of teachers in the same way that a superintendent may in a smaller city where all of the teachers know all the phases of work and so feel freer to express their opinions.

The rural school is organized with one teacher for all grades and for all subjects, with many classes to be heard daily and with long periods between recitations for each individual pupil. The county superintendent, with large territory to cover and with bad roads and inclement weather to work against, sees the teacher very seldom; the directors are inclined to visit the school still less frequently than does the county superintendent; so that the rural teacher is truly absolute monarch in his dominion, and, if he profits by his own mistakes, there is no better training for any teacher than just this independent battling with the problems of the professions.

A gla e at the second class of schools, those not supported by public funds, reveals very few schools which contain grades one to six. In the United States comparatively few parents send either boys or or girls away from home to school before they are 12 or 14 years of age, so that most preparatory departments in academies, colleges, etc., begin with work ranking in difficulty with that done somewhere from the sixth to the eighth grade. Some technical schools, trade and night schools, a few philanthropic schools like those of the Educational Alliance in New York and Hull House in Chicago, and some private schools like the Lower Shattuck (Faribault, Minn.), and the Horace Mann and Ethical Culture Schools of New York City, contain all the lower grades and deserve our study, but as these schools usually form no part of a "system," each stands alone in its organization, and consequently in its curriculum, both being largely based upon the aim which actuated the founding of the school.

TOPIC B: THE MATHEMATICAL CURRICULUM IN EACH TYPE OF SCHOOL.

Rural schools have the reputation of doing a greater amount of mathematics in grades one to six than other classes of schools, and though the studies made for this paper gave no proof of this, it may well be true, for such a result would be the natural outgrowth of (1) the old-time emphasis upon mathematics as a test of "brightness;" (2) the consequent selection of teachers (the "brightest" students of a community) good in mathematics, so that they probably taught this science better than they did other subjects, or, if not, they at least continued the emphasis upon it; and (3) the organization of the rural school, with its one teacher, many classes, and much necessary seat work or independent study. Since assignments in mathematics could be easily made and most easily corrected, since to have "ciphered through the book" was a mark of distinction, and since pupils who had any ambition to attain this distinction could thus be kept "in order" for long periods, mathematics came to form a major part of the curriculum of the rural school. Prof. W. S. Smiley's statement [1] that rural pupils are superior to urban pupils is of interest here, even

[1] Journal of Educational Psychology, vol. 1, no. 9.

though it deals with eighth-grade pupils, and probably contains some error in comparisons.

The graded school, with the necessity for meeting the wider and more varied interests of the city, naturally feels that other subjects must be given equal prominence with mathematics. The organization of the work usually permits one teacher to have a single grade, with two or at most three divisions in a room, so that long study periods are unnecessary. These two influences tend to make mathematics less prominent in city systems than in the rural districts. Changes in method, usually first adopted by city systems, also contribute to changes in the mathematical curriculum, so that we usually find graded-school curricula placing mathematical topics from one to two years farther up the grades than do rural schools. One influence that has been strongly felt in this postponement of difficult subject matter is that of the kindergarten, comparatively recently made a part of public primary education, it having begun as an independent factor. Before this, the tendency had been to push subjects lower and lower into the grades to meet the demands of the secondary school which in turn was dictated to by college and university. Out of the conflict which naturally ensued there has arisen much earnest thought and discussion over the mathematical curriculum of the common school, no decision as yet being reached. Nevertheless, through examination of courses of study and through the returns from some questions propounded to normal-school students, it appears that within a small range of variation the schools of the United States have fallen into a fairly uniform course of study in arithmetic.

A study of textbooks in common use shows a very marked change throughout the course in the sort of problems given as means of applying these fundamental operations. The latest and best arithmetics choose material closely related to daily life, so that children can not fail to feel a greater and more reasonable need of the mechanics of mathematics—a more "rational motivation" for the subject than pupils of the past could ever have felt.

TOPIC C: THE QUESTION OF EXAMINATIONS FROM THE POINT OF VIEW OF THE SCHOOL.

The writer's report on this topic is less valuable than on the others, because (1) the topic seemed difficult of interpretation, (2) the questionnaire given failed to limit the question to examinations in mathematics, and (3) the student's answers about time spent, etc., proved unusable.

To get the view of "the school" it was decided that one must know (1) the view of teachers, both from their own and their pupils' standpoints, and (2) the view of superintendent and supervisors whenever such view proved more representative of the "school's" attitude than

did the feeling of the teachers. Finally, the report is based upon very limited resources, viz: (*a*) The study of a few books or chapters dealing with the reasons for giving examinations; (*b*) an inspection of some examination questions given by different schools and State departments of education, (*c*) the giving of Dr. C. W. Stone's tests to a number of classes, and (*d*) such general information as a teacher naturally acquires during several years' experience.

Even to the present time examinations have been ranked most highly, but for different reasons by different schools. Some feel that the value lies in the mark obtained by the pupil, this mark being made a definite test of his ability, and upon it depending very largely his rank in class and his readiness for promotion. This belief causes great stress to be laid on "finals." Others deem the practice in organizing quickly, in thinking clearly, and in expressing accurately to be the main value of examinations. This theory tends to the use of monthly or biweekly examinations. Still other schools find the worth of examinations to lie not in a benefit to the pupils (save indirectly), but to the teacher, giving her, as they do, opportunities to check objectively her subjective ranking of pupils and of class progress. In this case, the examination papers are never returned to pupils, the teacher merely noting errors on which to work with both class and individuals. Yet other schools stand strongly for more or less frequent examinations, because of the chance this gives the superintendent to check the results obtained by different teachers, and to locate the cause of any unusual divergence from the norm, whether the cause lie in the teacher, in the pupil body, or in the course of study. Other reasons are advanced, but these are sufficient to illustrate their variance.

The present attitude and seeming future tendency concerning examinations is to rank them as valuable, but to let the value, and the consequent use made of the papers, be controlled wholly by the aim. The benefit to the pupils arises largely through practice (1) in separating essentials from nonessentials; (2) in clarifying a viewpoint through the enforced expression of it, or perhaps through its defense; (3) in learning to interpret the printed or written page independently, i. e., without the teacher's voice or eye or gesture to guide; or (4) in merely being tested upon memorized facts, which, though not the most vital thing in education, are yet the basis for all comparing, relating, and judging that are to follow. With this aim the tests should be frequent, should be carefully looked over by the teacher, and very soon thereafter should be made the basis of further class or individual instruction.

One proof that the first three forms of practice are growing in importance to teachers is found in the very different sorts of questions asked to-day from those of a decade or more ago. The earlier purely

memoriter questions are giving place to those which require old knowledge, not as isolated bits, but as knowledge to be utilized in new situations, to be marshaled quickly at will, and arranged logically, i. e., to questions which demand thought, judgment, choice, defense of opinion, etc.

Those tests, examinations, or parts of examinations whose purpose is to help the teacher check her judgment of pupils and test the portions of subject matter taught as to their relative value in giving the pupil ideals of and practice in persistence, honesty, independent thought-power, etc., need not be returned to the pupils. Though the teacher seems by this means to be saved considerable labor in checking errors, attempting to grade each paper justly, etc., the conscientious teacher has still the task of noting errors and classifying them in various ways; for example, errors due to ignorance or to carelessness; those relatively important or unimportant; class or individual errors; errors to be immediately dealt with or those which may better be handled each time an opportunity offers and so must be kept constantly in mind.

Were the correcting not so burdensome, many teachers would increase rather than decrease the frequency of examinations, especially if the time could be set and the questions made out by the teachers themselves. The greatest complaint about examinations comes from teachers who feel strongly that questions made out by any other person than the teacher of the children tend to become of the mechanical or memoriter type. Particularly is this resented when questions have so become, and when no latitude is given either pupils or teachers, i. e., when no account (in a large system) is taken of the personal equation, and when teachers and pupils are measured by these tests alone. As superintendents and supervisors make broader studies in these fields, they recognize the need of allowing for individual differences and the cause of the above complaint is fast disappearing. Without doubt a movement in the right direction is being made by Dr. Stone and others who are working out standardized tests, and it is hoped that this work will continue, being constantly improved and made more far-reaching in its effect. That superintendents find the returns as valuable in checking up their teachers as teachers do in checking up their pupils must be recognized by all.

To summarize, then, little change is seen in the value placed upon examinations; but marked changes are occurring in the type of the questions. The formal, lengthy, memoriter type which counted much in promotion and class ranking is rapidly changing to the type which is only another means of teaching, another form of recitation, a test similar to that which life daily demands of us, viz. to know some things perfectly, to relate these things to new situations, and thus quickly to solve new problems.

TOPIC D: THE METHODS EMPLOYED IN TEACHING MATHEMATICS.

In no other field under discussion can tendencies be more clearly noted than in this one of methods. Though many methods are still in use, there has been within a decade a most decided veering from mechanical rule-of-thumb methods to thought methods.

The present chaotic state of our methods in mathematics seems due to a number of causes, some of which are (1) the various views of what number is; (2) difference of opinion as to what shall be selected from the whole field to be taught in grades 1 to 6 of the elementary school; (3) the bondage we are in to past ideals; (4) the inertia of the school itself or the slowness with which a great institution like the school changes; (5) recent marked progress in the industrial world, demanding different life preparation of elementary school graduates; (6) social progress; and (7), though by no means least, the great demand for teachers, a demand so urgent that we press into service vast numbers of immature girls who, though earnest and zealous, yet lack that higher and broader professional training spoken of by Münsterberg which makes a teacher see the aims of education and know well the means available for meeting these aims.

The many methods which these causes produce may, after all, be classed into two main divisions—mechanical and thought—though it is true that most methods are a decided mixture of the two.

By mechanical methods we mean such as emphasize the symbol, the form, the expression, to the subordination, or even exclusion, of the thought, the content, the meaning. In direct opposition to this, thought methods, while not ignoring expression, lay greatest stress upon meaning.

Though the various views of what number is give a very different content to the number symbols in common use—that is, 2 may mean how many, how much, location in a series, the ratio of one magnitude to another, etc., according to the individual's view of the essentials of the number idea—yet, thought methods ever give this content first place, whereas mechanical methods center about the symbol. It is to be noted, however, that the human mind tends to make automatic, mechanical, or formal whatever it can. That symbols, then, incline to supersede meaning is due to very natural causes: (1) All must admit (*a*) that conventionalized symbols are needed to insure race progress, (*b*) that the higher the degree of conventionality the further is the symbol removed from its meaning, and, consequently, (*c*) that with this greater conventionality there is increased danger that the child may never get the meaning, that the symbols will grow to have undue importance to him so that he may, indeed, become a mere juggler with figures; and (2) if it is admitted that even any part of the science of number deals not with things but with relations of things, it must readily be seen what difficulty children have

in giving symbols any real content. Therefore has the teaching of the past, in a more or less conscious effort to meet children's needs, vacillated from too abstract teaching to teaching so concrete as to hide relations, or, in other words, from no attempt to make children understand, from avoiding content and giving rules and form to be mechanically learned, to such profuse and such long continued use of objects in trying to give a content to number that thinking has been retarded.

But that, in spite of our keeping away from either extreme, there is still an inevitable tendency among pupils to become formal even in what seems real thought work was proved satisfactorily to a group of observers who gave Dr. C. W. Stone's tests to one class of pupils, and, a month later, gave the same children the thought-test problems which they had not done, at the blackboard where work could be watched and where there was a chance to question the pupils. We were convinced that many of what seemed like thought processes really went off mechanically when the cue was at hand. Additional problems were given to the children, among them, this: A man hitches up a horse to a buggy and drives three hours at the rate of six miles an hour. How much farther would he have gone if he had hitched up two horses? All but one child doubled the distance. This child thought "no farther." Questioning this one child brought out from the others that the two horses would help one another so that some more distance would be covered, but certainly not double the distance. Another problem tried with many more individuals was the following (changed, in different cases, to sheep or dog on two or on four legs): If a duck weighs three pounds when it is standing on one leg, what does it weigh when standing on two? It is the exception to receive any other answer than twice the original number of pounds.

Though this remarked tendency to cast even the thinking into habitual forms must be watched throughout the early years of school life, it yet must be agreed that life demands that we learn many processes automatically. The error of the past was that these processes were the beginning and the end of the teaching. A more thorough study of method in all fields shows us that men have ever passed through three stages in the development of any field: (1) Simple or natural art, or mere manner of doing the deed; (2) conscious attention to the method, running into science; and (3) art again, finished art this time.

Methods which are predominantly thought methods plan (1) that pupils begin with content (having first felt some sensible reason for approaching the subject); (2) that they then pass to a use of symbols, to be handled automatically when expediency demands it, employing a particular form of expression only because that form best expresses

the thought held; (3) that pupils are encouraged in flexibility of expression as well as of thinking, the former, however, always being controlled by the latter; and (4) that pupils are given many opportunities to exercise choice and judgment in applying the knowledge gained to life situations.

Games, plays, and construction work are some of the means employed by many grade teachers to give content to, or else a rational motive for, many phases of number work, e. g., number combinations, United States money, and fraction study.

During the current year the writer has witnessed the teaching of some arithmetic lessons dealing with (1) finding areas, perimeters, etc.; (2) subtraction; (3) areas of triangles; (4) fractions; (5) commercial discount; (6) interest; and (7) insurance. The methods employed were mainly thought methods, for each time such an appeal was made to pupils' acquaintance with life situations as to make the plan in common use a most sensible one to the child, a plan he might even think out by himself, and never have given to him as a rule to be learned. The symbols or forms of expression grew from the children's hunting the best expression to say what they thought, and though they were plainly guided into adopting conventional modes of expression, the pressure was ever from the thought side.

Modern psychology, which emphasizes the formative as well as the revealing function of expression, enables us to select our methods of procedure as well as our mathematical curriculum with much more reason than did our predecessors in education. Though in the main content should precede form, there will ever be found situations which demand form before content, or if not before content, at least without content at the time of use. When thought methods are in the main employed in the presentation of such work the teacher is first sure that she has the content herself, then she presents the process in a way to make pupils recognize its rationality, even though they can not and perhaps should not be asked to explain; drill follows, and yet demands on thinking are made throughout; applications continue the thought work, but the wise teacher patiently waits for maturity to bring full interpretation. (Long multiplication might be cited as an illustration of such work.)

Finally, methods of teaching arithmetic, besides depending upon the subject matter chosen by makers of the curriculum, vary according to the aim of teaching mathematics held by the teacher, the school, or the community—whether the end of it all is technical skill for utility or economic purposes, whether it is formal discipline or culture, whether it is to interpret the quantitative side of life experiences, or whether it is a union of all these aims. Furthermore, the still more specific method selected for each recitation in mathematics

is determined by the narrow result desired or the means at hand for realizing this end, and hence often is, or may be, (1) objective, concrete, and rationally motivated, if a new topic is to be developed; (2) abstract and mechanical, if drill or mere repetition is the object; and (3) objective again, and full of original thought work, when application is the motive.

In conclusion, then, it appears that (1) the aims of teaching mathematics, the selection of subject matter, and the methods employed are constantly acting and reacting upon one another, so that a study of all is necessary to an understanding of one; (2) that topics in arithmetic for grades one to six are being rationally motivated, and processes are being steadily rationalized to the child; and (3) that thought methods are gaining ground.

II. THE ORGANIZATION OF SCHOOLS AND THE GENERAL RELATION OF EACH KIND OF SCHOOL TO THE OTHERS.

A. EXTERNAL ORGANIZATION OF THE ELEMENTARY SCHOOL.

The present organization of the elementary school is very difficult to describe, because of the many factors that have contributed to its upbuilding and the great variation that exists. We have no central school organization in the United States; each State is supreme in educational matters. The work of the United States Bureau of Education is advisory, instructive, and inspirational. It collects and publishes statistics in regard to educational matters at home and abroad; it publishes many fine articles and documents; but its greatest power lies in the way it unifies and shapes the educational thought of the country. School legislation is in the hands of the lawmaking body of each State, and the executive authority is vested in a State department. The officers of the State department are State superintendent and State board of education. The duties of the State superintendent vary in the different States, but in general, he has charge of the certification of teachers, State institute department, collection of statistics, the making of a course of study for the rural schools, publishing of bulletins on teaching, publishing of an annual report in regard to the work of the schools, and the apportioning of school funds of the State. In the States where the office of State superintendent has been dignified by giving it large opportunities and supreme authority in carrying out school matters the position is one of great honor and the educational affairs of those States are on a high plane.

The State board of education varies in its powers and functions. In some of the States it has entire charge of the normal-school system (Michigan, Illinois, Massachusetts, and Connecticut), while in others

its duties are conducting examinations and certificating teachers (California and Kansas). In the States where the board is made up of educators rather than politicians the influence is marked.

The political division of the State is the county, and the educational organization has the same unit. The county officers and the county organization vary sectionally. In the South the county is the smallest political unit, while in the North the county is divided into towns or townships, and in most States the townships are divided into school districts. The reason for this difference is historical and industrial. The southern colonies had the county unit probably because of the method of holding land. The large plantations gave a thin and scattered population, and thus a large area was necessary for a unit of government. Elementary education in the southern colonies was largely in the hands of private schools, and not until after the War of 1861 did State organization of school systems become universal. The county following the political division is generally the unit of school organization at the South. Louisiana calls the county the parish.

The county school organization is in the hands of a county superintendent or county commissioners and a county board, sometimes called board of examiners, in all of the States except those of New England. The office of county superintendent is generally an elective one,[1] and the duties vary from mere clerical work to real supervision of school organization and classroom teaching. "The county superintendent of schools shall have the general superintendence of the public schools in his county, except those in the cities which are organized under special law and those in special or independent school districts, and shall visit each public school under his supervision. He shall at such visits carefully observe the conditions of the school, the mental and moral instruction given, the methods of teaching employed by the teacher, the teacher's ability, and the progress of the pupils. He shall advise and direct the teacher in regard to the instruction, classification, government, and discipline of the school and the course of study. He shall keep a record of such visits, and by memoranda indicate his judgment of the teacher's ability to teach and govern, and the condition and progress of the school, which shall be open to inspection by any school director,"[2] etc.

The county board of education in many States together with the county superintendent has charge of the examination of teachers for county certificates.

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[1] Exceptions: Pennsylvania, township trustees elect county superintendent. New Jersey, Mississippi, Virginia, State board of education appoints county superintendent. Delaware, Florida, governor appoints county superintendent. Alabama, State superintendent appoints county superintendent. North Carolina, county board of education, clerk of supreme court, register of deeds appoint county superintendent.

[2] General school law of North Dakota, 1909.

In the South, where the county is the educational unit, the duties of the executive officers are quite different. The county organization of schools in Kentucky is quite typical, and is as follows: "Each and every county in this Commonwealth shall compose one school district * * * and shall be confided to the control of a county board of education. The county superintendent of schools, county judge, and county attorney of each county of the Commonwealth is hereby created a commission to divide their respective counties into four districts as nearly equal in area as is practicable. * * * Each educational district elects, at its regular November election, one member of the county board of education. These four members, together with the county superintendent, comprise the board of education, and have entire charge of the schools of the county." [1]

"The township organization of schools is but a merger of districts, with practically the same rights and privileges as were invested in each of the original districts. The officers are chosen at the annual town meetings by all the electors, or, as is sometimes the case, the township is the unit of school government and the schools are made uniform throughout its extent. Maine, New Hampshire, Vermont, Massachusetts, New Jersey, and Indiana have a compulsory township organization by legislative enactment. In at least 21 other States there is permissive legislation." [2]

The district is the smallest unit of organization and is found in some form or other in a large number of the States. This unit grew up in New England. The Puritan's spirit of determination to provide education for his children and the inconvenience of sending pupils a long distance to school, gave us the district schoolhouse in each settlement. The families sending their children to the school constituted the school district. Its size was a matter of convenience. When the New Englanders migrated to the West they took their school unit with them, making it more definite in organization and more suitable to the land holdings in size. The district to-day is a definite division of the township and is presided over by three officers elected by the voters of the district at an annual meeting. The annual school meeting is the most perfect democracy we have, as it elects its own officers, fixes the sum of money to be raised, and votes the number of months of excess of the school year prescribed by law. The term of school office is generally three years, and the officers have power to employ teachers, handle the funds of the district, and provide for the care of all school property.

The great industrial changes in this country, the large movement of the rural population to the cities, and the settling of large numbers of the foreign immigrants in the cities, have made the district organi-

[1] General school law of Kentucky, 1908.
[2] History of education in the United States, Dexter.

zation in many regions unsuitable. The large number of district schools with less than ten pupils has meant great expense, poor facilities for work, and unsupervised teaching. The State superintendents are urging upon the people the reorganization of the district and the consolidation of weak districts. The township unit is probably the unit of the future. Massachusetts solved this problem as early as 1882, and the other New England States have followed her example. "The school district in all towns in the state are abolished. * * * Every town shall choose by ballot at its annual meeting a superintending school committee of three. The management of the schools and the custody and care shall devolve upon the school committee, which shall annually elect a superintendent of schools, who shall not be a member of the committee."[1] This law does not apply to towns authorized to control school matters according to special conditions as in large cities. The Maine system is quite typical of township organization derived from the district organization. "The school committee of two or more towns having under their care and custody an aggregate of not less than 20 or more than 50 schools may unite in the employment of a superintendent of schools, provided they have been so authorized by a vote of their own towns at a regular town meeting or a special town meeting organized for that purpose."[2]

Every State has made special laws for the organization of city schools, and in many cases special charters have been given certain cities to meet unusual conditions. The city unit is usually a collection of districts. It is governed by a board of education and a superintendent. Other than this there is little uniformity of organization. The city boards of education vary in number from 3 members to 84, but a very common number is 9. The superintendent is elected by the board of education and is its executive in all matters pertaining to the workings of the school.

B. INTERNAL ORGANIZATION OF THE ELEMENTARY SCHOOL.

The term elementary school has various meanings. In the Eastern States it refers to the first nine school years or grades; in the South it usually means seven grades; and in the North and West eight grades. These grades or years are usually divided into four primary grades and four grammar grades. There are, however, other organizations, as in Tennessee, where the grades are defined by law, as follows:

The course of study in the primary school shall consist of five grades and the course of study in the secondary school shall consist of eight grades.

[1] Laws of Maine relating to public schools, 1909.
[2] Report of public instruction, New Hampshire, 1908.

* * * Pupils completing the first-year grades and attaining proficiency therein shall receive a certificate from the State superintendent certifying that the holder has completed the primary-school course, which shall be countersigned by the county superintendent and district directors and the teacher or teachers of the school, and shall entitle the holder to enter the sixth grade of the secondary school of any school district, or of the high school of any high-school district. The county high schools shall be graded by the board of education under the general regulation of the State superintendent and supervision of the county superintendent, beginning with the sixth grade, shall be adjusted for the admission of pupils who have completed the five grades of the primary schools.[1]

The coordination of the elementary school with the secondary school has been perfected in many States by a county or State leaving examination. This examination is given by the county board of education and all papers are rated by the county board. The examinations are the same throughout the county. A pupil receiving an elementary-school diploma of this character is allowed to enter any high school of the State without further examination. In California the law is as follows:

The courses of study for the day elementary schools of California shall embrace eight years of instruction * * * except in city school districts having boards of education, the county or city and county board of education shall provide for the conferring of diplomas of graduation by examination upon the pupils who have satisfactorily completed the course of study provided for the day and evening schools of the country or city and country.[2]

The States that have provided for the coordination of the elementary school with the high have raised their rural schools to a higher plane. The teachers who do not succeed in preparing their last classes for these county examinations soon find that there are no schools needing their services. Many schools look forward to the elementary-school graduation with as much pleasure as to the high-school graduation.

Some of our best educators are recommending a six-year elementary school period, to be followed by a six-year secondary school. This plan has many arguments in its favor. The pupil at the close of the sixth grade should be between 12 and 13 years of age. Those that are under grade average about 14. The under-grade pupils are in general the ones who leave school as soon as the age limit of compulsory education is reached. Nearly all of the States make the age limit 14 years. It would be an incentive to good work and regular attendance for the pupil to feel that graduation and a diploma are within his reach. From 12 to 14 there is a marked change in the ability of pupils to think and to grasp the meaning of problems. The teacher who is a specialist in his line of work appeals to the pupil. This is shown in the marked success of the departmental plan for the grammar grades. Looking toward a six-year high school

[1] Public school law, of Tennessee, 1909.
[2] School laws of California, 1909.

a number of our schools already have a seven-grade elementary school and a five-year high school. The Horace Mann School of New York City finds the latter an admirable scheme of organization.

C. EFFECT OF THE ABOVE UPON THE TEACHING OF ARITHMETIC.

This report is particularly interested in the effects of the organization of the schools upon the teaching of arithmetic. "There is not complete agreement among teachers as to the extent and character of the number work of the first school year."[1] All schools, rural and village, agree in general as to the amount of arithmetic to be covered in the grades, but the grade where formal arithmetic should begin, the way arithmetic should be taught to the young child, and the grade where arithmetic should be completed, are unsettled. "For the first half year the work should be mainly oral and largely incidental to other school exercises, especially to the construction work."[1]

"There should be no formal number work in this grade. In connection with the general oral lessons and occupation work many facts concerning the simpler number relations are informally presented and acquired."[2]

"Arithmetic grows out of other school exercises and is applied in problems of construction."[3]

"In grade one the children repeatedly discover numerical relations through counting. In grade two numerical relations are further revealed through work with measures. Grade three is a year of drill in number facts, the teacher resorting to the use of measures only when necessary. Grades four, five, and six are drill years in the processes and operations, with an increasing amount of applied arithmetic. In grades seven and eight pupils should increase in facility through abstract drill and in power to reason through applied arithmetic."[4]

In the course of study prepared for the rural schools of Michigan the entire work of the first four grades is oral, and the work of the first two places the emphasis on sense training. In the course of study prepared for the District of Columbia the first year is given to sense training. In the schools of San Francisco all of the work of the first grade is oral. It includes counting, addition, and subtraction, with nine as the limiting number. In the training school of the Michigan State Normal College the first year of the arithmetic course is devoted to sense training. "Not to put arithmetic as a topic in the first grade is to make sure that it will not be seriously or systematically taught in nine-tenths of the schools of the country.

[1] Course of study for the common schools of Illinois, 1907.
[2] Course of study, Denver public schools.
[3] Course of study, Cleveland public schools.
[4] Course of study, Boston public schools.

The average teacher, not in the cities merely, but throughout the country generally, will simply touch upon it in the most perfunctory way. Whatever of scientific statistics we have show that this is true and that children so taught are not, when they enter the intermediate grades, as well prepared in arithmetic as those who have studied the subject from the first grade on.

"Furthermore, while it is true that the essential part of arithmetic can be taught in about three years, it can not for psychological reasons be as well retained if taught for only a short period. The individual needs prolonged experience with number facts to impress them thoroughly on the mind."[1]

In regard to the completion of arithmetic there are three positions taken: First, that arithmetic should continue through the eight grades; second, that arithmetic should be completed in the seventh grade and algebra or geometry given in the eighth grade; and third, the simultaneous teaching of arithmetic and algebra or geometry in the eighth grade. The majority of schools teach arithmetic through the seventh and eighth grades but introduce the use of the equation in the fifth grade and give in the seventh and eighth grades some descriptive geometry in connection with mensuration. This course seems to make the necessary preparation for high school work.

D. ELEMENTARY PRIVATE SCHOOLS.

The private elementary schools are few in number. They are generally organized in connection with the private secondary schools as the elementary department. The private schools of the country are largely located in the East and South. Their purpose determines the organization and course of study. "The school aims to offer educational facilities fully abreast of the changing conditions of modern life; to retain those features of scholastic work that experience has stamped as sound and effective; to inaugurate promptly such plans as local conditions may require."[2] "The purpose of the school is to maintain a high order of training—mental, moral, and physical—to both boys and girls of the primary, intermediate, and high school grades.[3]

"The object of the school is to give thorough preparation for colleges and universities and at the same time to give instruction in those branches of an English education necessary to success in the ordinary pursuits of life."[4]

"The formation of character and not the acquisition of knowledge as an end in itself is the chief purpose of the school—a purpose

[1] David Eugene Smith: The teaching of arithmetic. Teachers College, Columbia University Press.
[2] Friends School, Baltimore, Md.
[3] Friends School, Washington, D. C.
[4] University School, Montgomery, Ala.

which the home and school should pursue together, in close cooperation with each other."[1]

"The title 'University School' partly explains itself, for the school's first aim is to thoroughly prepare boys for college work. The second particular purpose of the school is physical and moral development in connection with mental work."[2]

"The end set up is a social, an ethical one. The means taken to attain this aim are: First, the inculcation of the democratic spirit. * * * Children of the rich and poor and of different nationalities and races are to meet together and learn to respect one another, both in their work and their play; secondly, the awakening of serious intellectual interests and enthusiasms in order to counterbalance the pleasure-loving and self-indulgent tendencies; third, the awakening of the spirit of social service by enlisting the interest of the students in the work of settlements and neighborhood houses with which the school is in touch; fourth, the building up of a largely humanitarian and ideal purpose toward which all lines converge."[3]

And thus to give the aim and organization of the private school is to write all the individual means of carrying out the various aims. In general, the organization and course of study are the same as in the public elementary school. In place of emphasis each school shows its own individuality in carrying out the purpose for which it was organized. The variation in amount of arithmetic taught and time given is quite similar to that of the public school.

III. THE MATHEMATICAL CURRICULUM IN EACH TYPE OF SCHOOL.

Information concerning the course of study for the first six years of the elementary schools was obtained from the replies to a questionnaire sent to all States and leading cities and from an examination of a large number of printed courses of study.

Of the 48 States and Territories to which questionnaires were sent, 40 were heard from either directly through the State superintendent or from a representative city within the State. The eight from whom no replies came were in all cases States of small population. Of the 25 largest cities, 24 replied; of the 50 largest, 37 replied. The total number of cities heard from was 52. The total number of questionnaires sent out was 200; the number of usable replies, 90.

The replies from the leading private schools were limited and, as far as they could be interpreted, differed but little from those of the public schools. The replies from the State superintendents were inadequate owing to the impossibility of making generalizations upon the work of a State where there is of necessity, with few excep-

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[1] Francis W. Parker School, Chicago, Ill.
[2] University School, Cleveland, Ohio.
[3] Ethical Culture School, New York City.

tions, great variability in the courses of study. The most valuable replies were those from the leading cities of the country. A statistical report was therefore made from these. The 52 city school systems which formed the basis for the work represented in the aggregate 2,480,000 pupils. Every section of the country was represented, but the most densely populated States, such as New York, Pennsylvania, Ohio, and Massachusetts, were proportionately largely represented.

The questionnaire was in three sections. The first section asked for the means employed to broaden the scope of arithmetic, the second for means used to narrow the field, and the third asked for specific information as to the time the study of number was commenced and the year in which different arithmetical topics were studied.

The questions as to the means employed to broaden the scope were: (1) Are geometric forms studied? (2) Are the equation and a literal notation used in the solution of problems? (3) Is the application of number to manual training emphasized? To geography and nature study? To practical affairs? (4) What other means are used to broaden the scope of arithmetic?

In reply to the question as to the means used to broaden the scope of arithmetic through the study of geometric forms 72 per cent of the school systems replied in the affirmative, 23 per cent were in a qualified affirmative, such as "somewhat," "a little," making a total affirmative of 95 per cent and a total negative of 5 per cent. The questions on the equation and literal notation were, it was evident, understood to include seventh and eighth grades instead of the grades within the limits of the investigation. The replies, although of little value for this report, were as follows: Affirmative for the use of the equation, 51 per cent; qualified affirmative, 36 per cent; negative, 13 per cent; affirmative for the use of a literal notation, 9 per cent; qualified affirmative, 50 per cent (all labeled for seventh and eighth grades); negative, 41 per cent. In reply to the question on the application of number (1) to manual training, 50 per cent of the school systems replied in the affirmative; 35 per cent with a qualified affirmative "somewhat," making a total affirmative of 85 per cent and a negative of 15 per cent; (2) to geography and nature study, affirmative, 50 per cent; qualified affirmative, 18 per cent; total affirmative, 68 per cent; negative, 32 per cent; (3) to practical affairs, affirmative, 95 per cent; negative, 5 per cent. Typical replies to the fourth question as to other means used to broaden the scope of arithmetic were: "Our tendency is to concrete work in all grades in terms of a child's experience;" "Data for exercises taken from actual measurements and actual affairs;" "Practical affairs cover the ground;" "Arithmetic throughout is considered a social study;" "Use of actual tax bills, gas bills, water bills, etc. (forms borrowed

from public service companies); " "Arithmetic not valued as formerly for its worth in discipline, but it is being made very practical;" "Connected with home life and local industries."

The questions on the means used to narrow the field were: (1) Do you have the children do more work with small numbers and less with numbers in the millions than formerly? (2) Do you teach only those arithmetic topics for which children have immediate use, excluding such topics as interest and wall papering? (3) Other tendencies?

Of the replies as to the use of smaller numbers 91 per cent were affirmative. 3 per cent qualified affirmative, making a total affirmative of 94 per cent and a negative of 6 per cent. In reply to the question on the choice of those subjects for which the children had immediate use, 44 per cent replied in the affirmative, 32 per cent gave a qualified affirmative, making a total of 76 per cent affirmative and 24 per cent negative. The replies to the third question "Other tendencies" ran as follows: "Toward simplification," "Our tendency is toward the elimination of topics not clearly serviceable," "We emphasize mental and oral work with small numbers to secure facility in common processes," "Insistence upon simple problems and proficiency within narrow limits," "Toward simplification, accuracy; certainty," "Much work with small numbers; simplified problems," "The elimination of topics; emphasis on mental work."

In the third section of the questionnaire the following questions were asked: In what year is the study of arithmetic commenced? The four fundamental processes completed? The study of fractions commenced? The study of fractions emphasized? Ratio studied? Percentage commenced?

The replies answering these specific questions as to the order of work were as follows: Number was recorded as commenced in first grade in 71½ per cent of the school systems replying; in second grade in 22 per cent of the schools and in third grade in 6½ per cent. The four fundamental processes were reported as completed in the third grade in 5 per cent of the schools replying; in fourth grade in 78 per cent of the schools; and in fifth grade in 17 per cent. Fractions were commenced in the first grade in 14 per cent of the schools replying, in second grade in 21 per cent, in third grade in 17 per cent, in fourth grade in 21 per cent, and in fifth grade in 27 per cent. Fractions were emphasized in the third grade in 2 per cent of the schools replying, in fourth grade in 10 per cent of the schools, in fifth grade in 63 per cent, in sixth grade in 23 per cent, in seventh grade in 2 per cent. Ratio was studied somewhere in the lower grades as a basis for later work in 40 per cent of the schools, in the middle grades (fourth, fifth, and sixth) in 30 per cent of schools and left entirely for upper grades (seventh, eighth, and ninth) in 30 per cent.

of the schools. Percentage was commenced in the fourth grade in 5 per cent of the schools, in fifth grade in 30 per cent of the schools, in sixth grade in 45 per cent, and in seventh grade in 20 per cent.

It is difficult to get at the real truth in any situation through a system of questions and answers for the reason that almost any question which can be formulated admits of misinterpretation. By comparison of the replies to the questionnaire with the printed courses of study it was found that the greatest liability to misinterpretation lay in the first section of the questionnaire, which refers to the means used to broaden the scope of arithmetic. The replies throughout the section included in many cases the seventh and eighth years of the elementary school. This makes it necessary to interpret not only the answers in the equation and literal notation in a different way from which the replies indicated (attention has already been called to this), but the answers also on the application of arithmetic to manual training, geography, and nature study. In the second section of the questionnaire there was no difficulty. In the third section there was some difference in the interpretation of the question on the year in which number was commenced. In several schools where number was introduced incidentally in the first grade it was sometimes recorded as commenced there and sometimes not. The report, therefore, gives an idea of an earlier commencement of the formal study of arithmetic than actually exists. There was a similar confusion between incidental and formal work in fractions and ratio. Much incidental work in fractions is done in the first three years, but the study of a fraction as a fraction is left in the majority of cases to the fourth and fifth years. Ratio forms the basis of several special methods of teaching number used in many localities throughout the country. In such cases it is recorded as studied in the primary grades. Ratio and proportion as such are usually put down in the printed courses of study as work for the seventh and eighth years.

The summary is indicative of an advance in the work in general. One step forward is the tendency to broaden the scope of arithmetic so that it is less a series of exercises for the manipulation of figures and more of an introduction to mathematics in general. If we may take the summary as typical of the state of affairs, the study of geometric forms is becoming current, and in the seventh and eighth grades the use of the equation and of literal notation is making some headway. It is possibly wise that the latter are not used below these years, owing to the fact that the substitution of a formal means of solving a problem for a child's own natural logic is likely to cause confusion in his mind. The tendency to correlate arithmetic with other subjects and make it a thoroughly practical subject is strong. An equally striking demand throughout the country is the cry for the simplification of arithmetic through the use of smaller

numbers, the elimination of topics, the simplification of problems, and an emphasis on mental rather than written work. The tendency is to limit the work to what comes within the child's experience and expect greater efficiency within the narrower field. As to the program of work there is a tendency to begin number somewhat later in the primary school than formerly; to devote the years before the fifth grade to the four fundamental processes with integers, the fifth year to fractions, and the sixth year to fractions and percentage.

Since the cities as a usual thing are in advance of the smaller communities and the rural districts and serve, more or less as their leaders, the summary is more indicative of the trend of the courses of study than of the state of things as they actually exist. It shows a more advanced state of affairs than would be found extant if an average could be made of the work of the country as an entirety.

IV. THE QUESTION OF EXAMINATIONS FROM THE POINT OF VIEW OF THE SCHOOL.

The specific purpose of this section is to treat the question of examinations from the point of view of the school in its present status and as to its present tendencies.

The main sources of data on which this report is based are: (1) the returns from the questionnaire, formulated in conjunction with the subcommittee on the nature of promotion in elementary schools and admission to secondary schools, and sent out by the United States Commissioner of Education, and (2) the subcommittee's knowledge of the uses being made of "standardized tests."

Of the 1,000 questionnaires returned, 427 were fully tabulated. These were the first received and are believed to be representative.

The subject as assigned placed the emphasis on examinations as used and viewed by immediate school workers rather than by superintendents or examining boards. Hence the superintendents were requested to have the questions answered by their principals. This was done so as to make the replies represent more fully "the point of view of the school." Answers from teachers would have represented this point of view still more fully, but the securing and handling of a sufficient number to be representative of the entire country seemed impracticable.

Conclusions based on returns from questionnaires must always be made guardedly. Caution is especially necessary in dealing with these returns because of the unavoidable vagueness and overlapping of some of the answers. However, the tabulated answers seem to warrant the following conclusions:

1. The use of examinations is very general, practically universal. They are made use of to some extent in all grades above the second; 102 of the 427 schools report their use in the second grade; twice as

many in the third; and in the fourth, fifth, and sixth grades practically all report them.

2. In this very general use of examinations the point of view of the school is commonly recognized. This is clearly shown in the answers to the question, By whom are the questions made out? Only 106 of the 427 reports show the superintendent making examination questions independently; 220 show the teachers and superintendents or supervisors in cooperation; 203 show the teacher making questions independently; only 48 principals are reported as making the questions; and in only 22 answers were there any State or county questions reported.

3. There is no agreement as to the frequency of giving examinations. One hundred and forty-two say every month or less; 124 irregularly; 55 bimonthly; 48 quarterly; 180 twice per year; and only 19 annually. One conclusion seems safe, viz., that the practice of giving "finals" at the end of the year is decidedly uncommon. However, the 180 giving them twice a year may indicate some additional use as "term end" finals.

4. The use of examinations seems to be decreasing somewhat. Two hundred and fifty-six say it is decreasing, while 110 say increasing; 97 report no change; 57 do not know; and 19 say that the nature of examinations is changing.

5. The purposes for which examinations are given are not clearly indicated. There were only 99 of the 427 that gave "aid in determining promotion or giving marks" as a purpose, while 181 gave the answer, "to help the teacher judge the efficiency of her teaching;" and 390 "to test the knowledge or power of the pupils." While the vagueness of the last answer makes it susceptible to several interpretations, taking the 390 with the 181, it is probably safe to infer that the largest single purpose of examinations is to measure progress of pupils.

6. There seems to be a clearly voiced desire for "standardized tests;" 289 of the 427 answered "Yes" to the question, "Do you believe that there is any increasing desire for standardized tests?" and 73 said that such tests would be desirable if available; only 92 answered "No" and 22 "Not desirable." Those who understand the idea of such tests are enthusiastic in their approval.

The questionnaire answers gave evidence that standardized tests are not widely known; that they are comparatively little used; but that there is a present tendency to do so, since several representative schools are making use of tests recently published in book form.[1] These tests are standardized in that, under identical circumstances, they have been used to measure the abilities of pupils in the second

[1] Arithmetical Abilities, by C. W. Stone, Teachers College Bureau of Publications, Columbia University.

half of the sixth school year in representative systems of schools; and in that the respective achievements of these pupils are given in the book. By using these tests as directed anyone can determine where his pupils stand in the essentials of arithmetic as compared with the pupils of some of the best systems in the United States.

Accounts of four of the uses of these standardized tests have recently been published. The most recent of these articles is by Mr. J. C. Brown, of the Horace Mann High School, New York City, who used the "fundamental" test as a basis for determining the value of drill.[1]

Another recent investigation in which the tests were used was made by Prof. W. S. Smiley at the State University of Iowa. Prof. Smiley measured and compared the arithmetical abilities of one room of rural school pupils and graded town school pupils.[2]

Messrs. Birckhead and Somers, principals of schools at Louisa, Va., and Red Hill, Va., respectively, used the tests to measure arithmetical abilities in six representative public schools of Virginia.[3]

The most extensive study utilizing these tests is that made by Mr. S. A. Courtis, of the Detroit Home and Day School, a private school for girls in Detroit, Mich. Mr. Courtis is using the tests as a means of determining the effects of certain changes in the teaching of the mathematics for which he is responsible. Mr. Courtis has written a number of articles on the subject; he says in conclusion:

> More than all else, it has proved conclusively to the writer at least, that it is practicable to measure not only the general condition of arithmetic teaching throughout a school, the growth in ability and efficiency from grade to grade, the defects and needs of any one grade or individual, but the effects of changes in method or procedure as well.[4]

The value of standardized tests has appealed so strongly to Mr. Courtis that he has recently published a set that are by far the best extant;[5] and mathematics teaching will doubtless receive even greater help from him in the near future.

Another use of these tests may be illustrated by tests recently given to determine the effects of changes in teaching. When tested in 1907 the pupils of a certain school made a score of 468 points per 100 pupils in reasoning, and 2,311 points per 100 pupils in fundamentals. In 1910 the pupils of this same system made a score of 607 in reasoning, a gain of 139, and a score of 2,833 in fundamentals, a gain of 522.

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[1] An investigation of the value of drill work in the fundamental operations. Journal of Educational Psychology, February, 1911.

[2] A comparative study of the results obtained in instruction in the "single teacher" rural schools and the graded town schools. Elementary School Teacher (University of Chicago Press), January, 1911.

[3] How do our Virginia schools stand in arithmetic? Virginia Journal of education (Richmond, Va.), March, 1910.

[4] Elementary School Teacher, October and December, 1909, and December, 1910.

[5] The Courtis standard tests in arithmetic with manual of instructions for giving and scoring. S. A. Courtis, 441 John R. Street, Detroit, Mich.

Thus the use of these tests indicate that in this school the present teaching of arithmetic produces considerable more ability in the essentials than did the teaching of 1907.

Another purpose for which these tests are being given is to measure the effect of such radical changes in administration as omitting all arithmetic work from grades one and two. This experiment has been tried in several cities in New England, in a few on the Pacific coast, and in one in the Middle West.

The question that immediately arises is, How well do children learn arithmetic without its being taught during the first two grades? This question is being answered by measuring the abilities of children so taught and comparing them with the achievements of children who have had arithmetic teaching during the first two grades. In one of the above-mentioned systems the last class of pupils in the second half of the sixth grade, who had had arithmetic teaching in grades one and two, were measured. During the present school year the first class of pupils who have not had arithmetic teaching in grades one and two will reach the second half of the sixth grade. The abilities of this class will be measured and the results of measuring these two sets of pupils from the same system, together with the results from the original twenty-six systems, will constitute the basis of a comparative study which may yield objective data for answering questions concerning the advisability of omitting arithmetic teaching from the first two grades.[1]

The question of examinations from the point of view of the school may be summarized as follows:

1. Examinations are used in some form in some grades in practically all schools.

2. The point of view of the school is largely recognized.

3. As to the purpose of examinations and their frequency there is no agreement.

4. Their use is probably decreasing, especially the use of "finals," as the main basis of promotions.

5. There is increasing desire for "standardized tests."

6. These tests are beginning to be used to measure the relation between the theories and products of arithmetic teaching; and as a means of securing data from which the best procedures can be determined.

[1] Of course, the danger of fallacy here is evident. The abolition of arithmetic in grades one and two may have been accompanied by more enthusiastic and skillful teaching after the second grade, or by even a greater amount of time being devoted to the subject, so the results of such a comparison must be guarded by checks.

V. THE METHODS EMPLOYED IN TEACHING ELEMENTARY MATHEMATICS.[1]

I. SCOPE OF THE STUDY.

It is the function of this study to convey some notion of the methods employed in teaching mathematics in the first six grades of the American elementary school. No attempt is made to give a minute description of the endless details of teaching procedure, nor even to enumerate all the types of teaching method employed. Its purpose is restricted to an analysis of the larger tendencies in teaching practice which are representative of the spirit of mathematical instruction in the lower schools.

Owing to the existing confusions, it is well at the very outset to have in mind a clear definition of the term "teaching methods." Teaching methods are always methods of presentation. In this respect the teaching art is like any other art, literary, graphic, plastic, or what not. The literary artist, for example, has a purpose, a subject matter, a particular audience, and a special style of presentation. All these factors are present in the teaching art. The aims of instruction, the particular facts to be taught, the immaturity of the child taught, and the inevitable personality of the teacher determine the style of instruction, or, to use our own "trade word," a method of teaching. Every teacher, then, has a style or method—conscious or unconscious, good, bad, or indifferent. Unlike the literary artist, he has many ends to serve rather than one. His functions are general to life, and include moral, social, and personal ends, as well as those that are æsthetic. His methods of communication, too, are more than one. He presents his experiences objectively and graphically, as well as through the medium of written words and speech. Always the teacher's end is to stimulate growth through the presentation of experiences. When that presentation of experience takes a form and order different from that usual to adult life for the precise purpose of making the fact more readily comprehensible by the immature mind of the child, then that modification may be called a method of teaching. Teaching methods are always special manners of readjusting adult wisdom to the special psychological conditions of a student's mind.

In the concrete, methods of teaching would always represent something particular to a situation, and as variable as situations are variable. Life is never just the same at any point. Yet certain essential similarities exist and give us the opportunity to interpret life in terms of law. The same may be said of the teaching life. In a

[1] This part of the report was prepared by Prof. Henry Suzzallo, Teachers College, Columbia University, New York City. In more extended form it appeared in the Teachers College Record for March, 1911.

sense it never repeats itself, yet to the degree that the same end, the same subject matter, and the same immaturity of mind recur in classrooms, teachers will tend to use similar modes of adjustment. In describing mathematical teaching in the primary schools, it is these similar modes of teaching adjustment, these similar "general methods" that we shall describe and analyze.

It will be unnecessary to have a separate treatment of the "general methods" of mathematical teaching for public schools and private schools. Whatever may be said of the State-supported schools will in general be true of private institutions. It is true of elementary schools, as it is not of secondary and higher schools, that private institutions hold a relatively minor place, as compared with public or State schools. They are in a sense mere adjuncts to the public-school system, claiming, in the generality of cases, no real difference in their ideals and methods of instruction.

It will perhaps simplify the task of this study and make its treatment more thoroughly representative of all conditions, if the general methods described be restricted to that field which is most characteristic of the first five or six years of mathematical instruction, namely, to the teaching of the fundamental processes of manipulating integers and fractions along with their simple applications to concrete problems.

While the aspects of mathematical instruction here studied and presented are selected because of their representative nature, it would be unwise to restrict ourselves to a statement of the commonly accepted procedures of schoolroom practice. There are in America certain reform tendencies which are as characteristic of conditions as are the conservative practices. These modifying forces need to be mentioned along with the practices that they alter. Again, there are certain scientific efforts, now well underway, to study the problem of methods in teaching. While these have, as their immediate aim, the acquisition of new knowledge rather than direct educational reform, their ultimate effect will be to change methods of teaching. For this reason they are important, and have a proper place in this presentation.

II. THE INFLUENCE OF AIMS ON TEACHING.

The purposes of mathematical instruction in the elementary school must always be very influential upon method. It makes a great difference whether one is merely teaching the elements of mathematics or is teaching mathematics as a tool for business life.

THE INFLUENCE OF A SCIENTIFIC AIM.

It has not been long since the aim of mathematical teaching was merely scientific. The facts taught were the beginning of a science,

and the end was to obtain a foundation for more advanced facts of the same kind which were dependent upon this foundation. As the teacher had learned his mathematics, so he taught the subject. To a considerable degree as the master's adult mind classified the facts of the subject, so he presented it to the child. His methods were logical rather than psychological. He gave the finished product or process to the child without special modification because the child was immature; a roundabout method that slowly approximated and only finally achieved the full result was with such a teacher exceptional.

Such a scientific aim, implicit rather than expressed, dominated the methods of teaching when arithmetic was handed over to the elementary schools by the higher institutions of education. As the first purpose to be rooted in the traditions of mathematical teaching it still persists with all the rigidity of a conservative force. Teachers still tend to teach future workmen in the lower schools as they themselves were taught by scientific scholars in the universities. And high school and college instructors still impose their standards upon the lower schools so as to influence their methods of instruction. As the purposes of higher instruction still remain largely scientific in purpose and method they give aid to the persistence of an original tradition in the elementary schools. Under such an influence the worth of a mathematical fact is measured by its place in a logical scheme, rather than by its significance and recurrence in everyday life. The mathematician may need to know all about the names of the places in notation and numeration; the layman cares only about the accurate reading and writing of numbers, and not at all about the verbal title of "units of thousands" place. Again, the rational needs of a thinker about mathematics may require an understanding of the reasons why we "carry" in column addition, but the effective everyday use demands an accurate habit of "carrying" rather than an accurate explanation. Yet just such methods persist in our schools because of the domination of a scientific treatment of the subject.

The remoteness of such mathematical teaching from the needs of common life constantly threatens the loyalty and support of the public. Some defense becomes necessary on other than scientific grounds. Such a sanction could not be found in utilitarianism, for the waste was evident. It remained for a psychological theory to sketch a defense upon "disciplinary" grounds. The doctrine of "formal discipline" says that such mathematical teaching trains the powers of the mind so that any mastery gained in mathematics is a mastery operating in full elsewhere, regardless of the remoteness of the new situations from those in connection with which the power or ability was originally acquired.

The effect of such doctrine is to defend and perpetuate every obsolete, unimportant, and wasteful practice in the teaching of mathematics. No matter that partnership as taught in the schools had its original sanction in its close correspondence to the reality of business practice; no matter that the old sanction had passed; teach it now for its ability to discipline the mind. This questionable psychological doctrine, said in consequence, "Whatever is, is right!" No matter that "life insurance" touches more men than "cube root"; the latter should be kept because of its power to train the mind. In life, where "approximation" of amounts suffices, the teacher demands absolute accuracy, and the ethical worth of such precise truth is the high law for its defense. Regardless of the truth that is concealed in the doctrine of "formal discipline," it must be confessed by those who know the history of teaching methods in the United States that it is the main defense of conservatism and the largest cause of waste in teaching methods.

Such has been the ground upon which recent educational reform has operated. Slowly the older scientific and disciplinary aims of instruction have given way to the newer purposes of business utility and social insight. In that step a large transition was covered. Before the school had measured the worth of its work by standards internal to educational institutions, the schoolmaster and the scholar, rather than the man on the street, had formulated the scientific classifications of mathematics and expounded the doctrine of formal discipline. Hereafter the measure of efficient school instruction is a reference to standards external to the school, the product of conditions outside of school life. Business need and social situation determine if a fact or process is worth comprehending, and whether the method of instruction has been effective.

Business utility, coming at a time when the elementary-school course was felt to be overcrowded, met with a ready reception. It operated for the time being as the standard by which materials and methods in arithmetic are to be eliminated, if not actually selected. Materials not general to the business world, such as the table of troy weight, were therefore eliminated. Processes of computing interest infrequently used were supplemented by more widespread and up-to-date methods. More doing and less explaining characterized the instruction in adding columns of figures, and such manipulation mimicked the exact conditions of its use in the world at large. If strings of figures are usually added in vertical columns in the business world, then they should be taught in vertical columns more nearly exclusively than before. The obsolete and the relatively infrequent, the over-complex and the wasteful processes of the old arithmetic tended to disappear. More than any other influence this aim of business utility has combated the overconservative influence

of scientific and disciplinary aims which dominated previous decades. The newer methods of teaching have kept the best of the old movements. The work is still scientific in that it is accurate; it is still disciplinary in that it trains; but the truth and the training which are given are selected by and associated with actual business situations common to everyday life.

There is evidence in the present thought of teachers that a broader utility than that of the business world is beginning to obtain in the schools. The general increase in the social consciousness of the teacher is reflected in mathematical instruction. Everywhere in these days the American teacher and the educational writer speak of the social aims of education. The influence of the social aim of instruction upon mathematical instruction is subtle but obvious. The business man's opinion with reference to arithmetical instruction is not always taken as gospel. There are other standards. "Why," says the schoolmaster, "should I train people for your special needs, any more than for the demands of other trades that men ply? To be sure, our graduates do not fit perfectly into your shop at once. But that precise and local adjustment is the work of the business course or of shop apprenticeship. My function is to train men for the situation common to all men and special to no class. The elementary school is a school for general culture or social appreciation, not a business college or a trade school." The sociologist usurps the place of the business man as the school's proper critic.

The situation to-day as influenced by existing aims is one of transition in which old and new purposes mingle with unequal force in giving us a mixed process of instructing American youth in arithmetic. Old materials and methods still persist, for logical and disciplinary ideals still hold; but the newer regimen ushered in by the demands of business utility and social understanding gains ground. The obsolete, the untrue, the wasteful methods pass from arithmetic teaching, and the pressing, modern, and useful activities and understandings enter. Arithmetic is less abstract and formal as a subject than it was; it has become increasingly vital and concrete with real interests, insights, and situations. The grind of sheer mechanical drill decreases in its teaching, and a reasoned understanding of relations, in some degree, at least, is substituted. Artificial motives and incentives are less frequently used to get work done, and the appeal of live institutions for a quantitative solution and understanding increasingly asserts itself.

III. THE EFFECT OF THE CHANGING STATUS OF TEACHING METHOD.

Teaching method in the school is primarily a readjustment of forms of knowledge and experience so as more effectively to stimulate and improve the immature responses of children. Two important

movements have been responsible for the development of special methods of teaching during the past few decades—one is humanitarian and the other scientific. On the one hand, there has been a growth in reverence and sympathy for childhood. As yet it has scarcely expressed itself with fullness. The wide acceptance of the "doctrine of interest" in teaching; the enrichment of the curriculum; specialized schools for truants and defectives; individual instruction—these are the schoolmaster's recognition of the modern attitude toward childhood. Under such conditions teaching becomes less and less a ruthless external imposition of adult views, and more a means of sympathetic ministry to those inner needs of child life which make for desirable qualities of character. While it is true that teaching method is a condescension to childhood, it is a socially profitable condescension in that it is a guarantee of more effective and enduring mastery of the life that is revealed at school. Since the child's acquisition tends the more to be part and parcel of his own life under such sympathetic teaching, the products of such instruction are enduring.

Such a humanitarian movement naturally called for knowledge of the child. The wisdom of common sense soon exhausts itself and more scientific data is demanded. Thus the "child study movement" came into existence. Since then, a saner psychological foundation has been laid for educational procedure, one which is criticising and reconstructing teaching method at every turn. Hitherto teaching methods had been improved fitfully through a crude empiricism. Now a body of general psychological knowledge, rich in its criticism of old methods and in its suggestion of new means of procedure, gives a scientific basis to teaching method.

The public elementary school teacher is conservative indeed who will deny that there is anything worthy in the notion of "method!" As a class, teachers have faith in the special professional technique which is included under the term. They are critical of the many abuses which have been committed in the name of method. Method can not be a substitute for scholarship. It can not be a "cut-and-dried" procedure indiscriminately or uniformly applied to classroom instruction. Like every other technical means, teaching method is subject to its own limitations and strengthens a fad which the average teacher recognizes.

In spite of the fact that the majority of elementary teachers keep reasonably sane on the problem of method in teaching, it must be admitted that a considerable proportion of teachers are inclined to be attracted by systems of method that greatly overemphasize a single element of procedure. The hold which the "Grube method" with its unnatural logical thoroughness and progression gained in this country two or three decades ago is scarcely explicable to-day.

Scarcely less baffling is the very large appeal made by a series of textbooks which laid the stress upon the acquisition of arithmetic through the idea of ratio and by means of measuring. Manual work as the source of arithmetical experiences is another special emphasis, which, like the others, has had its enthusiastic adherents. Again, it is "arithmetic without a pencil" or some other overextension of a legitimate local method into a "panacea" or "cure-all" which confronts us. The promulgation and acceptance of such unversatile and one-sided systems of teaching method are indicative of two defects in the professional equipment of teachers: (1) The lack of a clear, scientific notion as to the nature and function of teaching method, and (2) a lack of psychological insight into the varied nature of classroom situations. Untrained teachers we still have among us, and others, too, to whom a little knowledge is a dangerous thing. These are frequently carried away by the enthusiastic appeals of the reformer with a system far too simple to meet the complex needs of human nature. Our experiences seem to have sobered us somewhat, the increase of supervision has made responsible officers cautious, and increased professional intelligence has put a wholesome damper upon naïve and futile proposals to make teaching easy.

A more serious evil than that just mentioned is the tendency of the supervising staff to overprescribe specific methods for classroom teachers. Recently there has developed, more particularly in large city systems, a tendency to demand a uniform mode of teaching the same school subject throughout the city. This has been brought about by the prevalent tendencies of large school systems to centralize their authority and demand uniformity of procedure. The prime causes of this tendency are to be found (1) in the specialization of grade teaching, and the interdependence of one teacher on another; (2) in the mobility of the school population which involves considerable lost energy if teachers do not operate along similar lines.

The result of such imposed uniformity is a reduction of spontaneity in teaching. The process of instruction proceeds in a more or less mechanical fashion, the teacher working for bulk results by a persistent and general application of the methods laid down. That teaching, which at every moment tends to adjust itself skillfully to the changes of human doubt and interest, difficulty and success, discouragement and insight, now taking care of a whole group at once, now aiding an individual straggler, now resolutely following a prescribed lead, now pursuing a line of least resistance previously unsuspected, can not thrive under such conditions. It stifles teaching as a fine art and makes of it a mechanical business. Under these conditions only those activities which fit the machine routine can go on. Thus it happens that we memorize, cram, drill, and review, and soon the subtler processes of thinking and evaluating, which are the best fruit of education, cease to exist.

Fortunately the one-method system of teaching will soon belong to the past, and the imposition of uniform methods is beginning to lose ground even in our cities. For the most part, the common sense of teachers and the positive statements of our better theorists keep teaching methods in a position of useful status. Teaching methods should be infinitely variable as the conditions calling for their use are endlessly changeable. Not one method but many are necessary, for their function is supplementary rather than computive. No one method should be used with a preestablished rigidity; each must be flexible in its uses, so as to accomplish the varied work to be done. The teacher directly facing the intellectual and emotional crises of childhood is the best interpreter of conditions and the best chooser of the tools of workmanship. The supervisor may advise and may point out certain fundamental laws of growth and procedure; but the concrete method which is the application of these must be of the teacher's making.

Arithmetical teaching, like the instruction in other subjects, has suffered from these widespread ventures of teaching method. In this respect it has shared the common professional lot. But in addition it has had special difficulties and adventures of its own. We have now to note those special phases of teaching method which are peculiar and local to mathematical instruction.

IV. METHOD AS AFFECTED BY THE DISTRIBUTION AND ARRANGEMENT OF ARITHMETICAL WORK.

THE TENDENCY TOWARD SHORTENING THE TIME DISTRIBUTION.

Several decades ago arithmetic, as a formal subject, was begun in the first school year and continued throughout the grades to the last school year. This is no longer a characteristic condition, much less a uniform one. There have been forces operating to complete the subject of arithmetic prior to the eighth year, and to delay its first systematic presentation in the primary grades for a period varying from six months to two years.

The attempt to shorten the period of formal instruction in arithmetic has had its effects upon the methods of teaching as well as upon the arrangement of the course of study. The presence of a large number of children who leave school by the seventh year, the example of a varied European practice, the overcrowded curriculum—all these have combined to suggest a shortened treatment of arithmetic. Hence economy, through the elimination of obsolete and unimportant topics in the course of study and through better methods of instruction, has become a pressing matter. Its influence on method is obvious.

It has focused attention upon "teaching method" and given it an increasing importance in the eyes of mathematical teachers. Specifi-

cally, it has tended to reduce the amount of objective work, to eliminate the explanation or rationalization of processes which in life are done automatically; it has made teachers satisfied with teaching one manner of solution where before two or three were given; it has laid the emphasis upon utilizing old knowledge in new places, rather than on acquiring new means.

The by-product of this belief is that any arithmetic taught during these first few years should be taught "incidentally," as a chance accompaniment of their other studies. Only after one or two years of incidental work should the formal arithmetic instruction be given. This "incidental" method of teaching beginners is difficult to estimate. It has been so variously treated that a comparative measure of its worth is difficult to obtain. The contention that children who are taught incidentally for two years and systematically for two years more have at the end of four years of school life as good a command of arithmetic as those who have had a systematic course through four school years is difficult to substantiate or deny on scientific grounds. Sometimes "incidental" teaching required by the course of study becomes "systematic" in the hands of the teacher. Sometimes the two years of "systematic" teaching that follows the incidental teaching means more than two years, since the teachers, in order to catch up, give more time and emphasis to the subjects than the relative time allotment of any general schedule would seem to warrant. Such have been the facts frequently revealed by a classroom inspection that penetrates beyond the course of study, the time schedule, and the regulations of the school board.

In the lack of specific comparative measures of the worth of such methods of instruction, there is a growing conviction (1) that beginning school children are mature enough for the systematic study of all the arithmetic that the modern course of study would assign to these grades; (2) that considering the quantity and quality of their experiences they can think or reason quite as well as memorize; and (3) that what the school requires of the child can be better done in a responsible, systematic manner than by any haphazard system of incidental instruction.

These reactionary attitudes by no means imply a return to systematic teaching of arithmetic in the first two school years, nor to such formal methods as had previously been employed. Other grounds forbid. The crude, uninteresting memoriter methods of the past have gone for good. Objective work, plays, games, manual activities make arithmetical study easier and more efficient. Indeed, these newer methods have been a large factor in convincing teachers that children have the ability to master the first steps in arithmetic during the first two years.

There are other problems of method that are not so much concerned with the beginning of the study of arithmetic, or with the span of school life that the subject is supposed to cover. These deal with the arrangement of subtopics, within the course of study, or with the manner of progression from one aspect of arithmetical experience to another.

The methods that have been employed in the United States for the arrangement or ordering of topics within the course of study have varied considerably from time to time, but all these variations may be grouped around two types: (1) The "logical" types of arrangement and (2) the "psychological" types of arrangement. If the course of study proceeds primarily by units that are characteristic of the mathematics of a mature adult mind, the type may be said to be "logical." If the course of study proceeds primarily by units that are characteristic of the manner in which an immature child's mind approaches the subject, then the type may be said to be "psychological."

The older "logical" plans are thorough and definite in their demands: the teacher always knows just what he is about. But such a system of procedure is unnatural and remote from the child: it lacks appeal and motive. The child pursues the subject as a task laid down for him, not as an answer to his own curiosities or necessities. The newer psychological plans meet the different levels of child maturity effectively; they are nearer the natural order of acquiring knowledge. But it is not easy for the teachers to keep account of the work of their own, previous, or subsequent grades. Nor does the supervising official find it easy to locate responsibility for definite arithmetical subtopics. As an order of teaching it is psychologically natural but administratively ineffective.

The result is that to-day the two types of arrangement are modifying each other and giving a mixed method, partly logical and partly psychological. That line of least resistance in which the children study arithmetical facts and processes with greatest success is modified by definite demands that topics, e. g., addition, be mastered thoroughly "then and there." The method is partly "topical" and partly "spiral." The child in the second grade may have a little of all the fundamental processes, a few simple fractions, and United States money, but just there he will be held definitely responsible for a very considerable number of the addition combinations. The pupil may have had fractions in every grade, but the fifth grade will be responsible for a thorough and systematic mastery of the same. Such is the mixed method of arrangement which is to-day prevalent in American schools.

V. OBJECTIVE TEACHING.

The use of objects in teaching arithmetic is current in the elementary school. Particularly is this true in the lowest grades of the school, in primary work. It may be said that there is a very large quantity of objective teaching in the first year of school and that it decreases more or less gradually as the higher grades are approached. By the time the highest grammar grades are reached the use of objects has reached its minimum, the underlying assumption being that the use of objects has a teaching value that decreases as the maturity of the pupils increases. Current practice does not proceed far beyond the application of the simple and somewhat crude psychological statement that the youngest children must have much objective teaching, the older less, the oldest least of all.

Reform in the direction of a more refined and exact use of object teaching has already suggested itself in the treatment of fractions and mensuration, where, regardless of the increased maturity of the children studying these topics, a large amount of objective method is utilized. This is a considerable departure from the slight objective treatment of other arithmetical topics taught in the same grades. Such exceptional practices suggest that the novelty of an arithmetical topic is the condition calling for objective work in instruction. It is immaturity in a special subject or situation which determines the amount of basal objective work. The correlation is not with the age of the pupil but with his experience with the special problem or subject in hand. It is, of course, true that the less experienced the student is the greater the likelihood that any subject presented will be novel and strange. Only in this indirect manner does the novelty of subject matter coincide with mere youth as an essential principle in determining the need of objective presentation. The naïve assumption of the older enthusiastic reformers that objective work is a good thing psychologically, one of which the pupil can not have too much, is by no means the accepted view of the new reformer. With the latter, objective presentation is an excellent method at a given stage of immaturity in the special topic involved, but it may be uneconomical, even an obstacle to efficiency, if pushed beyond.

There is then a certain coincidence of the scientific criticism of the psychologist and of the common-sense criticism of the conservative teachers who look suspiciously upon a highly extended object teaching. The teachers, on grounds of experience, say that too much objective teaching is confusing and delays teaching. The psychological critics say it is unnecessary and wasteful. The result is that, in these later days, the distribution of objective work has changed somewhat. More subjects are developed in the higher grades through objective instruction than before. Perhaps no fewer subjects

in the lower grades are presented objectively, but the extent of objective treatment of each of these has undergone considerable curtailment.

The existing defects in objective teaching are not restricted to a false placing or distribution. The quality of teaching with the aid of objects is likewise open to serious criticism. Object teaching is a device, so successful as against prior nonobjective teaching that it has come to be a standard of instruction as well as a means. As long as objects—any convenient objects—are used, the teaching is regarded as good. Given such a sanction, the inevitable result is an indiscriminating use of objects. The process of objectifying tends not to be regulated by the needs of the child's thinking life; it is determined by the enthusiasm of the teacher and materials convenient for school use.

The first fact which asserts itself in observing objective teaching is the artificiality of the materials employed. Primary children count, add, etc., with things they will never be concerned with in life. Lentils, sticks, tablets and the like are the stock objective stuff of the schools, and to a considerable degree this will always be the case. Cheap and convenient material suitable for individual manipulation on the top of a school desk is not plentiful. But instances where better and more normal material has been used are frequent enough in the better schools to warrant the belief that more could be done in this direction in the average classroom. The "playing at store", the use of actual applications of the tables of weight and measures are cases that might be cited.

The materials used are not only more artificial than they need be, but they are too restricted in range. More forms of even the artificial material should be used, thus minimizing the danger of monotony.

Even the narrow range of materials in general use might be better employed than it is. There is, of course, a distinct tendency to vary the objects, merely because a child gets tired of it as a material. It is too frequently the case that the teacher will treat the fundamental addition combinations with one set of objects, e. g., lentils. In all the child's objective experience within that field there are two persistent associations—" lentils " and " the relation of addition." The accidental element has been emphasized as frequently as the essential one, and being concrete has had even a better chance to impress itself.

The nature of the materials proper to objective teaching has likewise been too narrowly interpreted. Objective teaching has meant almost exclusively instructing or developing through three-dimensional representation, whereas another form has been neglected, which for all the psychological purposes of education has as much worth as so-called objects, namely, use of such material as pictures. Such quasi-objective material has been little used by teachers save as it

appears in textbooks. There are, of course, obvious disadvantages to pictures and diagrams. The things represented in and by them are not capable of personal manipulation by the child in the ordinary sense. But they have a superiority all their own. They offer a wider, more natural, and more interesting range of concrete experiences.

There are other curious phases of narrowness in the current pedagogical interpretation as to what constitutes a concrete or objective experience. It will be noted that visual objects are the ones generally employed and that they are generally inanimate objects. Of late there has been some tendency to use hearing and touch in giving a concrete basis to teaching. Advantage is taken of the social plays of children and their games with things. Here the children themselves and their relations and acts are the experiences from which the numerical units are obtained. With some of the best teachers in the lowest grades it is no longer unusual to see children moving about in all sorts of play designed to add reality to and increase interest in number facts.

Inductive teaching has been one of several movements affecting objective teaching. The effort of teachers to escape the slavishness of mere memoriter methods and to approximate real thinking led to the introduction of inductive teaching. Necessarily objective teaching became more or less identified with the new movement and was influenced by it. So, it has been said of objective work in arithmetic, as it has been said of laboratory work in the sciences, that such instruction is a method of "discovery" or "rediscovery." Such an alliance has had its beneficial effects upon objective teaching; it has redeemed it from the aimless "observational work" of an earlier "objective study." But in the teaching of arithmetic, at any rate, it confused an objective mode of presentation with a scientific method of learning truth, two activities having a common logical basis, but not at all the same. Under the assumption that the developmental method is one of rediscovery, the tendency is to give the child as complete a range of concrete evidences as would be necessary on the part of the scientist in substantiating a new fact. The result is, that long after the child is convinced of the truth, say that 4 and 2 are 6, the teacher persists in giving further objective illustrations of the fact. The child loses interest in the somewhat monotonous continuance of objective manipulations, and the teacher has naturally wasted time and energy.

Another modern movement in teaching method which has had a conspicuous effect on objective teaching is the movement toward "self-activity" on the part of the child. The recent favor enjoyed by manual training, nature study, self-government, and other active phases of school life is an index of the general movement in mind.

Its influence has not only forced the introduction of new subjects; it has changed the manner of presenting the older subjects of the elementary curriculum. Arithmetic has responded along with the other subjects and an active use of objects by the children themselves is found in increased degree.

VI. THE USE OF METHODS OF RATIONALIZATION.

It is perfectly natural that, in shifting the teacher's attention from her own activities to those of the children, the interest of the child should be considered in increasing degree. If the child is to learn directly, with a maximum use of his initiative, it is absolutely essential that the teacher should provide some motive. This implies that the child is to be interested in some fundamental way in the activities in which he is to engage. Instead of thumbing the fundamental facts with his memory, in an artificial and effortful manner, "singsonging" the tables rhythmically, so as to make dull business less dull, the teacher begins at once to use the child's own life as the basis for instruction. The number story, the arithmetical game, playing at adult activities, constructive work, measuring, and other vital interests of the child and community life become increasingly the basis of instruction in number. Such is the pronounced tendency wherever the movement is away from the traditional rote-learning or drill.

Of course there is the slight tendency in American elementary schools where a soft and false interpretation of the doctrine of interest is gospel to teach only those things which can be taught in an interesting fashion. But this tendency is less operative in arithmetic than in other subjects. Here the logical interdependence of one arithmetical skill on another has quickly pointed the failure of such a haphazard mode of instruction.

There is, however, in "advanced", as well as in reactionary quarters, a revolt against the tendency to objectify, explain, or rationalize everything taught in arithmetic. On the whole it is a discriminating movement, for this opposition to "rationalization" in arithmetical teaching, and in favor of "memorization" or "habituation," bases its plea on rational grounds, mainly derived from the facts of modern psychology.

It is specifically opposed to explaining why "carrying" in addition, and "borrowing" in subtraction are right modes of procedure. These acts are to be taught as memory or habit, inasmuch as they are to be performed by that method forever after. To develop such processes rationally or to demand a reason for the procedure once it is acquired, is merely to stir up unnecessary trouble, trouble unprompted by any demands of actual efficiency.

A study of the actual arithmetical facts upon which this opposition expresses itself suggests the four following general principles as to the use of "rationalization" and "habituation" as methods of mastery: (1) Any fact or process which always recurs in the same identical manner, and occurs with sufficient frequency to be remembered, ought not to be "rationalized" for the pupil, but "habituated." The correct placing of partial products in the multiplication of two numbers of two or more figures is a specific case. (2) If a process does recur in the same manner, but is so little used in after life that any formal method of solution would be forgotten, then the teacher should "rationalize" it. The process of finding the square root of a number illustrates this series of facts. (3) If the process always does occur in the same manner, but with the frequency of its recurrence in doubt, the teacher should both "habituate" and "rationalize." The division of a fraction by a fraction is frequently taught both "mechanically" and "by thinking it out." (4) When a process or relation is likely to be expressed in a variable form, then the child must be taught to think through the relations involved, and should not be permitted to treat it mechanically through a mere act of habit or memory. All applied examples are to be dealt with in this manner, for such problems are of many types, and no two of the same type are ever quite alike. These laws will, of course, not be interpreted to mean that no reason is to be given a child in a process like "carrying" in addition. The reason is not essential to efficient mastery, but it may be given to add interest or to satisfy the specially curious.

VII. SPECIAL METHODS FOR OBTAINING ACCURACY, INDEPENDENCE, AND SPEED.

It is not alone the first stages in the acquisition of an arithmetical process which have received attention in the reorganization of teaching methods, though, to be sure, the problem of first presentations has in recent decades been given the most attention. More and more the American tendency is to watch every step in the learning process, to provide for all necessary transitions, and to safeguard against avoidable confusions. It might be suggested that constant intermediation of the teacher in the child's work at every step might destroy the pupil's initiative and independence. Apparently, however, those who are so deeply interested that the child should not be permitted to fall into the errors which unsupervised drill would convert into habit, are fully as cautious to provide steps for forcing the child to assume an increasing responsibility for his own work. The distinction made is that an over-early independence is as fatal to accurate and rapid mathematical work as an over-delayed dependence.

One of the specific controversies much argued in the primary school concerns the medium through which arithmetical examples and

problems shall be transmitted to young children. There are three typical ways in which a situation demanding arithmetical solution may be brought to the child's mind: (1) The situation when visible may be presented through itself; that is, objectively; (2) the situation may be described through the medium of spoken language, the teacher usually giving the dictation; (3) the situation may be conveyed through written language, as when the child reads from blackboard or text. Inasmuch as objects are a universal language, no difficulty arises through this basic method of presentation. It is when a language description of a situation is substituted for the situation itself that difficulty arises. The child might be able to solve the problem if he really understood the situation the language was meant to convey. Owing to the difficulty that primary children have in getting the thought out of language, it has been urged that problems in any unfamiliar field should be presented in the following order: (1) Objectively or graphically; (2) when the fundamental idea is grasped, through spoken language; and (3) after the type of situation is fairly familiar, through written or printed language. It is seriously urged by some teachers that no written presentation should be used in the first four grades. Such an extreme tendency would practically abolish the use of primary textbooks. There are many exceptional teachers who do not put a primary text in the hands of children at all. Such a tendency is increasing. Particularly is this true among primary teachers in the schools of the foreign quarters of large cities. Accurate communication through the English language is always more difficult here. Hence, the period of objective teaching is necessarily prolonged, dependence on the "number stories" told by the teacher increased, and the solution of written problems much longer delayed than elsewhere.

The situation is somewhat different, almost the opposite in fact, when "examples" rather than "problems" are presented, meaning by "example" a "problem" expressed through the use of mathematical signs. It is easier to present "examples" in written form on blackboard or in text than it is to dictate them orally. This obviates the necessity of holding the examples in mind during solution. The permanence of the visual presentation saves the restatement frequently necessary in oral presentation. Hence it is a common practice to supply the youngest children with mimeographed or written sheets of "examples." It is with older children or with younger children at a latter stage in the mastery of a typical difficulty, that oral presentation of examples is stressed. Then we have that type of work which is called "mental" or "silent" arithmetic.

There is some tendency toward the provision of better transitions from the objective presentation of applied problems to the symbolic presentation of abstract examples. The nature of such a transition

is scarcely reasoned out as so much psychological science, but is the accompaniment of a widening professional movement for the enlarged use of pictures, diagrams, number stories, and the like. A critical examination of the various means of presenting arithmetical situations would order them as follows in making the transition from objective concreteness to symbolic abstractions: (1) Objects, (2) pictures, (3) graphs, (4) the concrete imagery of words, (5) more abstract verbal presentations, (6) presentations through mathematical symbols. No such minuteness of adjustment is apparent in existing methods, though it might seem desirable in teaching young children. There are four typical ways in which the child does his work, the names of which are derived from the differentiating element: (1) The "silent" method, otherwise spoken of as "mental arithmetic," "arithmetic without a pencil," etc. (2) The "oral" method, where the child works aloud—that is, expresses his procedure step by step in speech. (3) The "written" method, where the child writes out in full his analysis and calculations. (4) The "mixed" method, where the child uses all three of the previously mentioned methods, in alternation, as necessary for ease and efficiency.

The worth of these four methods of work is necessarily variable. The rapidity of the "silent method" with simple figures is obvious. The "silent method" and the "mixed method" (which is more slow but more manageable with complex processes and calculations) are the two methods normally employed in social and business life. The purely "oral" and "written" methods, with their tendency toward analysis and calculation fully expressed in oral or written language, are highly artificial. They are valuable as school devices for revealing the action of the child's mind to the teacher so that the same may be corrected, guided, and generally controlled. The present tendency is toward an over-use of these methods and toward an under-use of the other two, more particularly the "mixed" method. It would seem that there is little conscious attempt to make certain that the child moves from full oral or written statements to the judicious application of the more natural "silent" and "mixed" methods. It may be that full oral and written statements of work have seriously hampered the right use of the more natural methods of statement.

It is well to recall that in all these efforts to control the child's activity there is a tendency to leave the child overdependent upon the teacher. It is vitally important that a child should be kept free of any error which unsupervised drill would fix into the stubbornness of habit, but it is likewise important that the child should acquire some self-reliance. While not always clearly defined, there is a distinct tendency in the direction of releasing the teacher's control of the child. A characteristic practice would be one in which the

teacher's work with the child would pass through various stages, each one of which would mark a decrease in the control of the process by the teacher and an increase in the freedom of the child to do his example, or problem, by himself.

One characteristic series of stages quite frequently used in the presentation of a single topic in arithmetic, say "carrying" in addition, is the following: (1) The teacher performs the process on the blackboard in the presence of the class, the children not being allowed to attempt the process by themselves until after the process is clearly understood from the teacher's development. (2) The children are then allowed to perform the process upon the blackboard, where it is exceedingly easy for the teacher to keep the work of every child under her eye. An error is caught by a quick glance at the board and immediately corrected before the child can reiterate a false impression. (3) More of the same type of example, or problem, are assigned to the children at their seats, where they work upon paper, still under the supervision of the teacher; a supervision which is less adequate, however. (4) The same difficulty, after the careful safeguarding of the previous stages, is then assigned for "home work," where the child relies almost completely upon himself. Once more it is necesary to suggest that these stages are merely roughly implied in the variations of existing practice.

Most of the methods discussed in this chapter have had as their sanction the attainment of accuracy in thinking and calculating. Some efforts to insure independent power on the part of the child have already been noted. But nothing has been said of the effort to add speed to accuracy in getting efficient results. Such special efforts have been made. These efforts may be classified into two groups: (1) Those aiming to quicken the rate of mental response. (2) Those aiming at short-cut processes of calculation.

Typical of the first are (*a*) the use of an established rhythm as the child attacks a column of additions; (*b*) the device of having children race for quick answers, having them raise their hands or stand when they have gotten the answer; (*c*) the assignment of a series of problems for written work under the pressure of a restricted time allotment for the performance of each. These and similar devices are much used in the schools. They are open to the objection that they quicken the rate of the better students, but foster confusion, error, and discouragement among the less able children, not infrequently, actually retarding speed.

The various shorter methods which represent the effort to reduce the number of mental processes required are usually not of general applicability, and consequently have not attained any general currency in the elementary schools, where the object is to teach one generally available and effective method, even though it requires more

time, special expertness being left to later development in the special school of business which requires it.

It has come to be quite a common recognition of teachers that the fundamental element in rapid arithmetical work is certain and accurate work. If pupils know their tables of combinations and are sure of each detail of calculation, there is no confusion or hesitancy; speed then follows as a matter of course. This belief, as much as anything else, explains why the lower schools have developed few special means for attaining speed apart from those already mentioned.

VIII. THE USE OF SPECIAL ALGORISMS, ORAL FORMS, AND WRITTEN ARRANGEMENTS.

The methods of teaching arithmetic are influenced not only by the aims of such instruction, but by the peculiar nature of the matter taught. The use of special algorisms, temporary algoristic aids or teaching "crutches," oral and written forms of analysis are of considerable moment in determining the difficulties and therefore the methods of teaching. Their condition and influence will need to be given some slight notice.

The use of special and temporary algoristic aids or learning "crutches" in mathematical calculation is one of the problems of method under constant controversy. Teachers seem fairly evenly divided upon the question. Typical situations in which such "crutches" are used may be noted as follows: Changing the figures of the upper number in "borrowing" in subtraction; rewriting figures in adding and subtracting fractions, in the broad sense any algorism which is used during the teaching or learning process temporarily, to be abandoned completely later, is an "accessory algorism" or "crutch." The objections to their use lie in the fact (1) that skill in manipulation is learned in connection with stages and forms not characteristic of final practical use; (2) that this implies, psychologically at any rate, the waste of learning two forms or usages instead of one; and (3) that it decreases the speed with which mathematical calculation is done. If there is a drift in any direction, it is probably toward the abandonment of "crutches."

The division of opinion, which exists in connection with the temporary use of special algorisms or "crutches," likewise exists with reference to the use of "full forms" and "short forms" of manipulation and statement. The temporary use of a "full form," in a case where a "short form" will finally be used, is similar to the employment of a "crutch." There is one important difference, however, which explains the relatively larger presence of temporary "full forms" than of "crutches." The "full form" is an accurate form which is used somewhere, in a more complex stage of the same

process or in some other process; the "crutch" is not. Thus, a "full form," in column addition (with partial totals and a final total of partial totals) will be utilized in column multiplication; the "long-division form" of doing "short division" (that is the fully expressed form of dividing by a number of one figure) will be utilized in division by numbers of more than one figure.

The problem of form applies not alone to the algorism or special method of computation, but it likewise applies to the special methods of reasoning used in determining the specific series of steps to be taken in achieving the answer. In every problem the child solves he must not only decide what is to be done (reason), but he must do it (calculate). There are forms of reasoning as there are forms of calculation. As any calculation may have several algorisms the solution of a problem may be expressed in several forms. It is the latter difficulty which appears in the teacher's demands for "formal analysis" of problems. The analysis is usually required in full statement.

A conservative protest against the old formal expression of reasoned steps is found in omitting for the most part the linguistic statements dealing with the logic of the problem and merely "labeling" the numbers that occur in the calculation. This is a more restricted form of statement, much more used at the present time than hitherto. But it is still open to psychological objections that make the more scientific critics protest. There are many stages in a calculation where there is no association whatever with the concrete problem in hand. The concrete problem is studied, the decision is made that all the factors named are to be added. They are added, purely abstractly, and a number is given as the total. The result is then thought of in terms of the concrete problem in hand. A disposition to label each item in the addition may be necessary in the rendering of a bill, but it is a false and obstructing activity in the actual solving of the problem. The same situation exists where there are two or three processes to be utilized in series. Once the child has grasped his concrete situation and reasoned what to do he may proceed to mechanical manipulation, never thinking of the concrete applications till he is done.

The same tendency which is making for a reduction of verbal forms is increasing the use of mathematical symbols. As logical relations are less frequently written out, a simple sign such as $+$ or $\div$ is used. The algebraic x is supplied in place of a whole roundabout series of awkward preliminary statements or assumptions. With it, of course, come changed methods of manipulation, as in the use of the algebraic equation.

It is doubtless true that the rigidity of full logical forms is giving way to a more flexible and natural mode of expressing the child's

thoughts. Fixed oral and written forms of exposition may assist the child, much as the acquisition of a definite symbol fixes an abstract meaning, which remains unwieldly until it attaches itself to a word by which it is to be recalled. But increasing care is manifested that children shall use only those forms that will conform to practical need upon the one hand, and to natural, efficient, and economy mastery on the other.

IX. EXAMPLES AND PROBLEMS.

The teaching of arithmetic is usually classified under two aspects, formal work and applied work. The formal work deals mainly with the memorization of fundamental facts, processes, and other details of manipulation. The applied work, as the name implies, is the formal work utilized in the setting of a concrete situation demanding a solution. These two aspects of arithmetical instruction are very frequently sharply separated, the child working alternately with one or the other. The characteristic practice is to deal with them without relating them as closely as the highest efficiency would demand.

Formal exercises in arithmetic are usually presented through the "example;" the exercises in application through the "problem;" the distinction being that one is an abstract and symbolical statement of numerical facts and the other a concrete and descriptive statement.[1] In the first case the mathematical sign tells the child what to do, whether to add, subtract, multiply, or divide; the example being a kind of prereasoned problem, the child has only to manipulate according to the sign, his whole attention throughout being focused on the formal calculation. In the second case the child has two distinct functions: He must, from the description of the situation presented, decide, through the process of reasoning, what he is to do (add, subtract, divide or multiply), and, having rendered his judgment, he must proceed through the formal calculation.

As the problem involves two types of mental processes in a single exercise, and the example but one, the usual procedure in arithmetic is to take up the formal side through examples first and, later on, the applied side through the use of problems. This means that the first emphasis is on formal and abstract work rather than on a treatment of natural, concrete situations, an emphasis not wholly sanctioned by modern psychology and the better teaching procedure of other subjects.

[1] While this distinction is not general, it has sufficient currency to warrant its use here for the convenience of discussion. The expression "clothed problem" (from the German) is occasionally used to mean what is here designated as "problem," and "abstract problem" is used to mean what is here designated, as "example."

The reform tendency is found mainly in the primary grades where the beginnings of new processes are made through objective presentations of the problem. But the transition from objectified problems to formal work is not immediate. The children pass from objectified situations to "number stories," which are only descriptions or narratives of a situation. This is the interesting primary-school equivalent of that more businesslike language-description found in the higher grades—the arithmetical problem. But it precedes formal work and succeeds it, the formal drill being a mere intermediate drill. Here concrete presentations and formal work are more closely related and more naturally ordered.

This reform tendency, which began in the primary school, is extending to the higher grades, where it is no longer rare to find the attack upon a process preceded by careful studies of the concrete circumstances in which the process is utilized. In the case of interest, several days might be utilized in studying the institution of banking in all its more important facts and relations. Such an approach not only provides motive for the formal and mechanical work, but it gives a necessary logical basis in facts. Hence, the understanding of practical business life makes accurate reasoning possible for the child when he is called upon to solve actual problems in application of the formal work.

It is perfectly natural under the general traditional practice of putting the first emphasis on mastery of the formal work that the largest amount of attention should be given to the mechanical and technical side of arithmetic, and that the concrete uses and applications should be slighted, and this is generally true of the practice of American teachers. Much more ingenuity has been used in the careful training of the child on the formal side than in teaching him to think out his problems. There is no such careful arranging and ordering of types in teaching a child to reason, as there is in teaching him to calculate.

Here and there a few thoroughly systematic attempts are made to carry the pupil through the simple types of one-step reasoning, to two-step and three-step problems with their possible variations. Just as the example isolates the difficulties of calculation, by letting the sign of + or − stand for the logic of the situation, there is a tendency to give problems without requiring the calculations. This affords a means of isolating and treating the special difficulties of reasoning. The child is merely required to tell what he would do, without doing it; the answer being checked by the gross facts. A little later, as a transition, he is permitted to give a rapid, rough approximation of what the answer is likely to be. With further command he tells what he would do and does it accurately. But such a program of teaching is still rare among teachers.

The care of the child's reasoning is largely restricted to testing his comprehension of the problem (1) by having him restate the problem to be sure he understands it, or (2) by having him give a formal oral or written analysis of the way in which he solves the problem. The first requirement may not be thoroughgoing, as the child may give a verbal repetition of the problem without really knowing its meaning. The second is a formal analysis of the finished result and does not represent the genetic method of the child's thinking. Consequently its formulas do not in any considerable degree assist him in his actual struggle with the complex of facts.

This lack of a systematic teaching of the technique of reasoning is manifest in the unreliability of children's thinking. When a child fails in a problem assigned from the textbook, the source of the error may be in one or more of three phases: (1) In failing to get the meaning of the language used to describe the details of the situation; (2) in failing to reason out what needs to be done to solve the situation; (3) in failing to make an accurate calculation. The first is a matter of language; the second, one of reasoning; the third, of memorization. The elimination of errors, due to the first and third sources, leaves a considerable proportion to be accounted for by the second. Such informal investigations as have been made seem to show that the children who fail in reasoning do not make any real effort to penetrate into the essential relations of the situation. They depend on their association of processes with specific words of relation used in the description of the problem, an association determined, of course, by their past experiences. As long as these familiar "cue" words are used, they succeed. Let unfamiliar words or phrases be utilized in their stead or let the relation be implied, and, like as not, the children will fail to do the right thing. Practical school people are familiar with the fact that children solve the problems given in the language of their own teachers and fail when the problems are set by principals or superintendents, whose language is strange to them.

A greater use of varied objects in the objective presentation of problems, and a more constantly varied use of language in the descriptive presentation of problems would prevent the child making such superficial and unthoughtful associations, and force him to think out connections between what is essential in a typical problem and the appropriate process of manipulating it. But such a deliberate application of modern psychology is far from being a conspicuous minority movement. The subject matter of the problems given to children has, however, improved greatly. Obsolete, puzzling, and unreal situations which only hinder the child's attempt to reason are less and less used in problem work.

Daily it becomes recognized with greater clearness that right reasoning depends upon a comprehension of the facts of the case, and the facts of the case in point must be within the experience of the child. This is the only way in which a problem can be real and concrete to him.

The recent effort on the part of textbook writers and teachers to make arithmetical problems "real" and "concrete" has not always recognized the above-mentioned psychological principle. The terms "real" and "concrete" have been interpreted in many ways, besides in terms of the child's consciousness. With some, "real" has meant "material," and the problems, more particularly with primary children, have, in increasing degree, been presented by objects or words connoting very vivid images. Others have defined these qualities in terms of actual existence or use in the larger social world. If these problems actually occur at the grocer's, the banker's, or the wholesaler's, it is said that they "are indeed concrete." And much effort has been expended in carrying these current problems into the classroom, in spite of the fact that they may be neither comprehensible nor interesting to the pupil.

There is another social world, nearer home to the child, from which a more vital borrowing can be made. There is an opportunity to use the child's life in its quantitative aspects, to take his plays, games, and occupations, and introduce their situations into his mathematical teaching. As his world expands from year to year he will be carried by degrees from personal and local situations to those of general interest. The teacher can provide this progression without devitalizing the facts presented.

There is another error into which both the socially minded radical and the specialist in child study fall. In their eagerness to improve the arithmetic problem, they assume that problems taken from the larger social world or from the child's experience are necessarily superior to hypothetical, imaginative, or "made-up" problems. The psychological fact that needs to be forced upon the attention of the reformers is that, with proper artfulness, an imagined problem may be even more vital and real to the child than one taken from life—as a situation in a drama may be more appealing and real to a child than one on the street. This has some recognition, but not enough. Those who stand upon the side of the "made-up" problems are more likely to be reactionaries who tolerate the traditional type of problem even though its stupid artificiality is obvious to both the teacher and the child. They might better be dealing with dull problems borrowed from real life than with specially invented dullness.

Of course there is another argument for the use of actual social materials. The child must ultimately come into command of precisely these facts, since their mastery will be demanded by the busi-

ness world. But must a primary school child study his arithmetic through problems taken from the dreary statistics of imports and exports merely because tariff reform is a political issue which every citizen ought finally to comprehend? There is a time for this, and, as is the case with most of such borrowed business problems, the time is later. In so far as these are current situations within the contacts of child life, let them enter. A quantitative revelation of life is important, and it is good teaching economy to gain knowledge by the way, provided it does not distract attention from whatever main business is at hand.

The socializing of arithmetical problems has one other additional good effect. It has tended to bring some topical unity into the problems constituting the assignment for a given lesson or group of lessons. Hitherto a series of problems was almost always composed of a heterogeneous lot of situations. There was no unity save that some one process was involved in each. The movement is now in the direction of attaining a more approximate unity within the subject matter of the problems themselves. The difficulties of attainment have restricted this movement to more progressive circles.

The eclectic source of arithmetic problems is apparent from the foregoing discussion. It would seem that some better texts would naturally be evolved through the implied criticism of each movement upon the other. Such is the case. Problems from child life emphasize the beginning condition to which adjustment must be made in all good teaching. Those from the greater world suggest the final goals of instruction. Those "made up" by the teacher call attention to what is too often forgotten, that the educative process in school may be artful without becoming artificial. Teaching is art, and when well done is not less effective for the fact.

X. CHARACTERISTIC MODES OF PROGRESS IN TEACHING METHODS.

The existing methods of teaching arithmetic in the American elementary schools are exceedingly varied. This is due to many causes. The democratic system of local control, as opposed to a centralized supervision of schools, has increased both the possibility and the probability of variation. Even within the units of supervision (State, county, and municipal) the opportunity for reducing variation in the direction of a more efficient uniformity is lost. This is partly due to the lack of a thoroughly trained staff of supervisors of the teaching process. Uniformity beyond the legal units of supervision has been restricted by the lack of organized professional means for investigation of and experimentation in controversial matters. Even such crude experiments as are being tried in more than one classroom, school, or system are unknown, unreported, unestimated, because no competent professional body gathers, evaluates, and dif-

fuses such knowledge. In this respect the teaching profession is far below the efficient organization of the legal and medical professions.

It is exceedingly difficult, therefore, to analyze the characteristic aspects of teaching method except as these are interpreted in movements of general significance. These may be actual or potential, traditional or reformatory, general or local, in present acceptance. The situation is one wherein tradition is mixed with radicalism, and radicalism modified by reaction. In this medley of movements there are dominant tendencies both traditional and progressive.

It is quite impossible to indicate the progressive tendencies with clearness save in connection with the discussions of concrete difficulties in mathematical teaching. The forces that are behind these tendencies may, however, be summarized here. For convenience they may be classified into eight types of influence, extending from more or less vague and general movements to very particular scientific contributions. No attempt is made to indicate the achievement of each; the form of each influence is only roughly defined and illustrative movements or studies suggested.

1. It is obvious that any general pedagogical movement which influences the professional attitude of teachers will influence the special methods of mathematical teaching. The appearance of the doctrine of interest made mathematical instruction less formal. The growing enthusiasm for objective work enlarged the use of objects in the arithmetic period. The child-study movement laid emphasis upon the child's own plays and games as a source of problems and examples.

2. Certain special movements in methods of teaching, local to the subject of mathematics, have also been effective. Here one has only to recall the "Grube" method, with its influence on the order and thoroughness with which the elements of arithmetic are taught.

3. The tendency of every teacher who is at all sensitive to the defects of his methods is to vary his daily practice. Constant trial, with error eliminating and success justifying a departure, is thus a source of progress. The new devices of one teacher are taken up by the eager professional witness and innovation is thus diffused. We have no ability to measure how much professional progress is due to individual variation in teaching and its conscious and unconscious imitation. The disposition of school systems to send their teachers on tours of visitation without loss of salary is a recognition of the value of this method of advance.

4. A far more efficient and radical source of change than that just mentioned is the deliberate, conscious, experimental teaching of progressive individuals. Some new idea or device occurs to the reader of original mind, and it is tried out with a fair proportion of resulting successes. An illustration of such a contribution is found in

one conspicuous effort to get more rapid column addition. The first columns to be added were allowed to determine the selection and order of addition combinations learned. Thus, if $6+7+9+6+7=35$, is the first column to be used, then the first combinations mastered will be $6+7=13$, $3+9=12$, $2+6=8$, $8+7=15$. Arising as a fruitful idea and seeming to give a measure of success, it has been carried, in the particular locality in mind, from school to school, and from system to system.

5. A prolific source of radical change is found in the critical application of modern psychology to teaching methods. Algorisms, types of difficulty, the order and gradation of these, as well as many other factors in method have been radically reorganized on psychological grounds. Examples of such psychological modifications of method are found in the "Courses of study for the day elementary schools of the city of San Francisco." Still more extensive critical applications are found in the "Exercises in arithmetic" devised by Dr. E. L. Thorndike, professor of educational psychology in the Teachers College, Columbia University.

6. Attempts have been made to inquire into the special psychology of arithmetical processes through careful experimentation and control. They have not been numerous, nor have they been influential on current practice. Such a field needs development. A typical attempt to investigate and formulate the special psychology of number is found in a Clark University study of "Number and its application psychologically considered." [1]

7. Educational investigations as to the efficiency of existing arithmetical teaching among school systems, sufficiently varied to be representative of American practice, have also been conducted. These have usually gone beyond the field of the special methods of presentation employed in the classroom, and have inquired into the conditions of administration and supervision, the arrangement of the courses of study, and other similar factors. Dr. J. M. Rice's studies into "The causes of success and failure in arithmetic" [2] investigated such specific factors as: The environment from which children come, their age, time allotment of the subject, period of school day given to arithmetic, arrangement of home work, standards, examinations, etc. A subsequent study of similar type, but employing more refined methods is that of Dr. C. W. Stone on "Arithmetical abilities and some factors determining them." [3] The main problem of this study was to find the correlation between types of arithmetical ability and

[1] Phillips, D. E. Number and its application psychologically considered. Pedagogical Seminary, vol. 5: 221–281, 1897–98.

[2] Rice, J. M. Educational research; Causes of success and failure in schools. Forum, 34: 281–297, 437–452, 1902–3.

[3] Stone, C. W. Arithmetical abilities ad some factors determining them. Columbia University contributions to education, Teachers College, New York City, 1908, pp. 101.

different time expenditures and courses of study. These two studies have probably attracted more general notice than any other studies of arithmetical instruction. While they have largely dealt with administrative conditions that limit teaching method, rather than with the details of teaching method itself, they have stimulated the impulse to investigate conditions and practices of every type.

8. The latest source of progress in teaching method is found in the movement for comparative experimental teaching under normal but carefully controlled conditions. Several such experiments are being conducted in the Horace Mann Elementary School of Teachers College, Columbia University, under the direction of Principal Henry C. Pearson, with the cooperation of the staff of Teachers College. The experimental work conducted by the instructors and students of the Teachers College is primarily designed to determine the relative value of competing methods in actual use throughout the country. the assumption being that every substantial difference in practice implies a difference of theory and consequently a controversy that can be resolved only on the basis of careful comparative tests. Two parallel series of classes of about the same age, ability, teacher equipment, etc., are selected for this work. One series is taught by one method; the other series by the other method. The abilities of these children is measured both before and after the teaching, and the growth compared. The standards and methods of this type of comparative experimentation, together with a list of current competitive methods requiring investigation, is given in Dr. David Eugene Smith's monograph on "The teaching of arithmetic." [1]

SUBCOMMITTEE 4. PREPARATION OF TEACHERS FOR GRADES ONE TO SIX.

INTRODUCTION.

The fact that mathematical instruction in elementary schools is so universally given by teachers who teach most of the other elementary school subjects also, makes it difficult to isolate many of the specific elements of preparation entering into training for mathematics teaching. Departmental instruction below the seventh grade in mathematics is so rare as to be exceptional. The general preparation for mathematics teaching in the first six grades of the elementary school is therefore very largely included in that general foundation work which includes courses in psychology, the principles of teaching, and the principles and history of education, together with observation and teaching under supervision and criticism in a training or

[1] Smith, D. E. The teaching of arithmetic. Chap. XVI, Teachers College, January, 1909.

practice school. Besides this general type of pedagogical preparation, most normal schools and other training schools for teachers usually offer courses in "special methods" in teaching elementary mathematics, these courses furnishing practically all of that training which may be regarded as specific for the subject. The "special methods" course is usually given by an instructor who also teaches classes in secondary mathematics—advanced arithmetic, algebra, geometry, and occasionally trigonometry, or even more advanced courses which are offered as electives in a very few normal schools. In the larger schools, the head of the department may be assisted in this instruction, but he, himself, usually gives the work in "methods." The character of this work and the professional attitude of the instructors vary in a degree that makes generalizations relative to them almost impossible. The instructor may regard the "methods course" as simply a means for reviewing subject matter and the training school work as a sort of nuisance or as having no relation to his department, or he may give the course over almost wholly to questions of the psychology and pedagogy of mathematics teaching, using the training school constantly as a laboratory for instruction, demonstration, and experimentation.

Until the rapid development of normal schools within the last quarter of a century, in common with all other elementary school subjects, the only preparation required for teaching mathematics in the elementary school was such a course in elementary arithmetic as would enable the candidate to pass a relatively simple examination set by a county or city superintendent or a board of school directors. Even to-day, in most of the rural and village communities of this country, this is the method of providing teachers. These teachers have no academic preparation beyond the subject matter which they expect to teach, and no professional training at all. However, the rapid development of State normal schools and of city training schools has brought about a standard for teachers, adopted as fast as the supply will permit, of much higher academic preparation together with a minimum approximating two years of professional training. In this general advance of standards, mathematics has received a share of attention, although perhaps tradition has offered a handicap which has not been felt by the newer subjects in such a large degree. But within the last decade the whole subject of the teaching of mathematics has come to receive the attention which it justly deserves. Yet, with all of the discussion which has developed, there is very little literature that deals specifically with the problem of present-day theory and practice in the preparation of teachers for the elementary school. The committee secured data for its conclusions by direct inquiry of typically representative normal schools and city training schools. By geographical distribution, 19.5 per

cent of these schools are in the Atlantic States, 21.9 per cent in the Pacific and Mountain States, and 58.5 per cent in the States lying between these groups. The inquiry concerned itself specifically with the amount required of prospective teachers in the subject matter of mathematics, the amount and kind of professional study, the relation of the department of mathematics to the training or practice school, and suggestions of constructive improvemnt or reform.

SUBJECT-MATTER COURSES.

For normal schools regarded as standard, as well as for city training schools, high-school graduation seems almost a fixed norm for the whole country as an entrance requirement. In mathematics it is usually assumed that students have had a high-school course in arithmetic of one term or semester, one year or more of algebra, and one year of geometry, usually plane. In some normal schools arithmetic is required of students "found wanting" in its subject matter. In many schools solid geometry, advanced arithmetic, advanced algebra, and trigonometry are offered as electives.

It is thus apparent that the teacher of mathematics in the first six grades has a knowledge of subject matter in formal mathematics far beyond any of the actual needs arising within these grades. That this knowledge of subjects beyond elementary arithmetic functions in the teaching of the arithmetic it would be very difficult to show. The formal and isolated character of the work in algebra and geometry, as they are usually taught, leaves them barren of any content values having a bearing upon anything which appears in the usual work of the first six grades. Even in the best normal schools there is little evidence that the work in algebra and geometry is any less academic than in the classical high schools. In the normal school "humanistic" values should certainly most fully reveal themselves.

PROFESSIONAL COURSES.

The professional courses and work include a course in arithmetic "methods" or in arithmetic with some attention to methods in all normal schools, observation and practice teaching in some form in most of them, and a course in the history of mathematics in 4.8 per cent of them.

The course in arithmetic "methods" is given during one semester in 78 per cent of the schools. In a few, not more than 10 per cent, it is given during but half a semester—from six to nine weeks. The character of this work differs markedly. In 27 per cent of the schools the work is largely a review of the subject matter of arithmetic, method being merely incidental. Over 14 per cent give practically the whole

time to method. With others it varies from one-tenth to one-half of the time.

In the methods work itself 41 per cent of the schools require the organization of definite units of subject matter for presentation in the grades and the writing of specific lesson plans as a part of the work. In a like number definite study is made of the elementary course of study as a whole. Games and recreational devices for motivation and drill are considered in 53 per cent of the schools. Forty-one per cent of the schools make some use of current literature on the teaching of mathematics, although one instructor replies that he does not, as "there is little in it that is useful." Other instructors indicate that they value the periodical literature as growing increasingly helpful and suggestive for their own work. Some form of mathematical museum is found in 34 per cent of the schools. This varies from "a small private collection" to rather pretentious collections of textbooks, periodical essays and addresses, illustrative apparatus and models, and standards of weights and measures of various kinds.

Observation of schoolroom teaching as a part of the work in methods is found in 46 per cent of the schools. The time devoted to observation varies from "three or four lessons a term" to "one-third of the course." In 7 per cent of the schools all observation work is in connection with practice teaching. The place and value of systematic, purposeful observation seem to vary greatly in the minds of instructors. That the lessons observed should be well taught, model lessons, is indicated by the fact that in 47.3 per cent of the schools definitely using observation the lessons are taught by critic teachers and in 36.8 per cent by the special mathematics teacher. The training school is used as a laboratory in the methods courses in 39 per cent of the schools. As schools of education which tend to furnish an increasing number of teachers prepared for normal-school instruction in mathematics develop possibilities of the training schools as laboratories, this phase of work in the preparation of the elementary-school teacher will doubtless increase.

Practice teaching in arithmetic is required of all students who graduate from 39 per cent of these schools. In one school it is required in at least two grades. In most other schools students do practice work in but two or three subjects and may be graduated without any practice teaching in mathematics.

The somewhat isolated condition of the department of mathematics from the training school is shown by the fact that in only 39 per cent of the schools does the teacher of mathematics aid in the supervision of the subject in the training school and that in only 27 per cent does the department of mathematics make or partly make the course of study for the training school. Cooperation in the development of the course of study and in the supervision of the teaching

of mathematics in the training school has been found to be of very high value in developing a professional attitude in the minds of prospective teachers through the multitude of live problems always before them and in whose solution they may participate.

More than 95 per cent of the schools discourage departmental work below the seventh grade. One school definitely advocates it from the fourth grade upward. A special type of training for primary teachers is provided in 24 per cent of the schools. Number work receives its proportion of attention in these primary methods courses when they are general in character. In 12 per cent of the schools there are specific courses in primary methods in number work.

From the suggestions for reform received from instructors, the following summary indicates the several types: "A better knowledge of subject matter." "Teaching to make subject matter vital." "Make the work closer to the children; more connection of the work with life." "Use more concrete examples; use more objective work." "Develop a more definite knowledge of practical applications." "Develop more work in the fundamental pedagogy of processes." "Develop a more definite idea of the relation of the work of each grade to the whole arithmetic course." "Develop a better knowledge of method." "The development of a good textbook in simple method." "Require successful practice teaching in arithmetic for at least 10 weeks." "Make more of observation; observe more skilled teachers." "A general course for all, followed by a more intensive course for the particular grades in which the teacher is to work."

SUMMARY OF DATA.

From the foregoing data, it is evident that great differences in ideals and practice exist with reference to the following points: There is variation from method as a mere incident to subject matter to the use of full time for method in "methods" courses; from no use whatever of current literature on the teaching of mathematics to "very extensive" use of such literature; from no consideration of games and recreational devices to very careful consideration and testing of these; from no consideration of the course of study in arithmetic for the grades to the full development of such a course and the organization for presentation of certain of its units; from no mention at all of the history of the development and teaching of mathematics to the establishment of well-organized courses in this subject; from no observation of lessons in arithmetic in the grades in connection with methods courses to one observation lesson each week during the course; from no practice teaching at all required in mathematics to practice work in arithmetic for all in at least two grades; from no supervision of practice work and the teaching of mathematics in

the training school by the teacher of mathematics to close, jointly responsible supervision with the grade supervisors; from positive discouraging of departmental teaching of mathematics in the grades to positive advocating of it in middle and upper grades; and from no differentiation in training for the grades to courses in detailed special methods for primary grades.

There are also evident the following points of general uniformity: Entrance requirements to methods courses in arithmetic are quite uniformly high-school courses of about one-half year's work in arithmetic, one year in algebra, and one-half year or one year in plane geometry. Some kind of methods or teacher's course in mathematics is found in all. Some form of observation work either in connection with the course in arithmetic methods or with general method or practice teaching is advocated by all. In general, departmental work below the seventh is discouraged. No school is fully satisfied with its present practice.

Suggestions for reform are of four types: Greater knowledge of subject matter; a reorganization of arithmetic material, giving it more vital relationship to the child's life and to social life; a more intelligent knowledge of the pedagogy of arithmetic; and a closer, more vital relationship between the department of mathematics and the teaching of arithmetic in the training school. Three ways are suggested for bringing about this last relationship: By more systematic observation of work in the grades, by more responsibility for the course of study and the methods of teaching in the grades, and by closer supervision of practice teaching in arithmetic by the department of mathematics.

CONSTRUCTIVE SUGGESTIONS.

From inferences based upon the foregoing, and from reflection upon the general problem, the committee believe that the best theory and practice of to-day point to the following conclusions: That a foundation in subject matter as a basis for the professional study of mathematics for teaching the subject in the first six grades of the elementary school should include a minimum of one-half year in high school arithmetic, one year of algebra, and one year of geometery; that, exclusive of all courses in psychology, pedagogy, principles of teaching, general method, or history of education, a minimum of one-half year's professional study of arithmetic should be required to include the following: A course in "special method," the teaching of elementary mathematics which should consider the more elementary phases of the psychology of number; principles of general method in their application to arithmetic; educational values of arithmetic and the place of arithmetic in the general educational

scheme; the organization of the elementary school curriculum in arithmetic; the organization of typical units of subject matter for presentation to appropriate grades; the development and writing of typical plans for teaching; the utilization of local and general economic studies for number application; the observation and discussion of typical lessons in the grades showing concrete applications of principles developed; the place of games and other recreational devices in lower grade work in number, and the historical development of the teaching of arithmetic, showing the place and value of certain "methods," as the Grube, Speer, etc.

The committee believe, further, that every school engaged in the preparation of teachers of mathematics should develop a museum or teaching collection of materials—apparatus, books, pamphlets, papers, etc.—which will aid in interpreting the historic development of the subject, present day practice, textbooks, etc.; that the head of the department of mathematics should be largely responsible for the organization of the course of study in mathematics in the training school in cooperation with the department of education and the supervisors in the training school; that the head of the department of mathematics should aid in the supervision of the teaching of mathematics in the training school; and that he, as well as the supervisors or critic teachers, should be able to give demonstration lessons in the training school illustrative of principles of teaching developed in the "methods" class, and that the points of emphasis in all observations, discussions, plans, and criticisms should be upon the basis of fundamental principles rather than upon petty details.

The aim in the whole professional consideration of mathematics for teachers of these grades is, broadly, to give acquaintance with the fundamental principles of teaching arithmetic, of the organization of its subject matter, of its place in the educational scheme, and of its historic development. The teacher should be given the pedagogical outlook and perspective of arithmetic, as well as the ways and means of teaching its details. He should know enough of the psychology of number to enable him to secure healthy interest and adequate drill and to sacrifice neither at the expense of the other.

The greatest problem of all at the present time would seem to be to find teachers for departments of mathematics in normal schools who themselves have the wide pedagogical outlook desired for such work. When teachers can be found who have its larger perspective and who will regard the training school as the laboratory for developing insight, intelligence, and a minimum of skill in the teacher to be sent out into the field at large, this vital, daily union of theory and practice will do much to increase the efficiency of the prospective teacher in elementary mathematics.

SUBCOMMITTEE 5. MATHEMATICS IN GRADES SEVEN AND EIGHT OF THE PUBLIC AND PAROCHIAL SCHOOLS.

SCOPE OF THIS REPORT.

It was desired that this subcommittee make a special study of the following topics for grades seven and eight in public schools and in parochial schools: (*a*) The organization of the schools. (*b*) The mathematical curriculum in each type of school. (*c*) The question of examinations from the point of view of the schools. (*d*) The methods employed in the teaching of mathematics. (*e*) The parochial schools.

It was desired that the report present both present conditions and tendencies.

SOURCES OF INFORMATION.

In addition to general knowledge of the ground covered by this report, use has been made of the following sources of information: (*a*) The teaching of elementary mathematics. Prof. David Eugene Smith, Macmillan Co. (*b*) The teaching of arithmetic. Prof. David Eugene Smith, Teachers College Press. (*c*) The teaching of mathematics. Prof. J. W. A. Young, Longmans, Green & Co. (*d*) Courses of study of leading cities. (*e*) Responses to a questionnaire sent to leading cities.

DEFINITIONS OF TERMS.

The term elementary education is probably not as well defined as it might be. In general it is that education which by common practice is deemed desirable and necessary for preparation for the duties and rights of citizenship. It is made compulsory as a rule by the laws of the various States for children between the ages of 6 and 14 years. The term public as applied to schools is used to denote those schools which are conducted at the expense of and under the control of the various State and local political units; tuition in these schools is without expense. For a more complete discussion of the units see the report of the general committee number one. The term "grade" is used to denote each of certain divisions into which the work of the elementary schools is divided. As a rule there are eight such divisions. One school year of from 35 to 40 weeks is required for a pupil to complete the work of one grade, excepting that bright pupils are permitted to do the work in less time. In larger cities a year or two of preliminary (kindergarten) work for children between the ages 4 and 6 is sometimes given. The eight main divisions are called respectively the first grade, second grade, etc. In some parts of this country, notably New England, there is a slightly different division of the course. The function of this committee is to study the mathematical work in the last two grades of the elementary course.

SPECIAL INTEREST IN THESE GRADES.

There are several reasons for special interest in the work of these grades. The mathematical function of the first six grades is to give the pupils control over the mechanics of arithmetic, over the fundamental number, facts and processses. The function of the upper grades has not been as well defined. There are several influences at work affecting the whole curriculum of these two grades, and especially the mathematical course. The results of certain psychological studies are especially applicable here; these two grades comprise the last part of the compulsory education of the children; they precede the high school; they correspond with certain sections of the school course of European schools, which are included in the secondary curriculum.

INFLUENCE OF PSYCHOLOGICAL STUDIES.

No effort will be made to go into great detail on this point; attention will be directed briefly to certain conclusions reached by psychologists, which are especially effective in modifying the mathematical work.

The doctrine of formal discipline is accepted now in only a modified form so that it is no longer deemed sufficient to claim for any subject that it has great disciplinary value. The educational value of all subjects, including mathematics, has been and is being subjected to close scrutiny with the result that subjects and subject matter long retained in the curriculum, through regard for tradition, are being displaced by new material, equally valuable as means of training, but more representative of current life. When this is not the result, old material is frequently dropped without any such substitution, on the ground that it neither has valuable content nor is necessary as means of training. As in all reforms, there is a tendency to go to extremes in this, especially with regard to mathematics. The willingness of the mathematicians to subject their science to the test of the new doctrine has invited the less sympathetic scrutiny of others who are not interested in the subject. The result is a tendency to demand more of mathematics in this respect than of other subjects.

The doctrine of interest as essential in training the will is generally accepted as valid. Systematic efforts to study child life and needs have served to call attention to the special interests of children between the ages of 12 and 15 years, who are just entering the adolescent period of life. The importance of safeguarding nervous energy at this time is generally recognized; the need of training the hand and the eye along with the mind, the necessity of vital contact with new material rather than theoretical study as a basis of real interest and comprehension are among the results of this movement.

INFLUENCE OF COMPLETION OF ELEMENTARY COURSE.

A large majority of the pupils of the schools do not go on to high school after completing the eighth grade. There are many reasons for this. The laws of most States do not permit the employment of children under 14 who have failed to complete the eighth grade. Many who are kept in school thereby avail themselves of the opportunity to seek employment as soon as they have completed the grade. In many cases the need of getting employment is urgent; in others, the opportunity to do so, the fact that the organization of the schools makes completion of the eighth grade a definite scholastic achievement, and the interests of pupils at this age lead many children to leave, even when their parents are in financial position to send them to high school. Many parents and pupils feel that the next stage of education is dominated too much by cultural ideals of education of so general a nature that the pupils are not trained for any form of remunerative employment. As evidence of this belief one may point to the large number of pupils who have graduated from the eighth grade who attend private schools where tuition is charged, called commercial schools, and to the fact that new forms of secondary education which have been provided in some localities seem to meet a real demand without in any way decreasing the demand for the customary forms of secondary education. For these various reasons there is a large shrinkage in the school population in the passage from the eighth grade to the high school.

Those in control of the educational work of the country are more and more allowing this fact to influence the course of study in the seventh and eighth grades. There is increasing emphasis on such training as will definitely equip the graduates for their future lives as individuals and as citizens of the community. Forms of manual training are provided and commonly, when it is impossible to furnish the equipment for all of the grades, provision is made for the pupils of the two upper grades. Owing to the realization that this vocational training is inadequate, there is at present a growing interest in industrial training. This movement which is destined to spread is certain to have its effect upon the mathematical work of the seventh and eighth grades.

INFLUENCE OF PREPARATION FOR HIGH SCHOOL.

It has been mentioned that many pupils who are ready for the high school fail to enter. It has been felt that some pupils might be led to go on to the high school if they could have in the eighth grade the beginnings of some of the high school subjects. For this reason, Latin, algebra, and a modified English course are urged as desirable

eighth grade work in the hope that pupils will be unwilling to discontinue these subjects after they have once started them. In more recent years there is a reaction against this plan on the ground that these subjects are not properly within the scope of elementary education as defined. At the present time there are the beginnings of a new administrative policy to accomplish the same end. Schools are being provided in some of the systems which are designed for pupils who do plan to go on to high school or whose parents wish them to have this relatively more academic training. In these schools algebra will easily find a place. It is likely that the future will see a growth of this idea.

Again, for various reasons, the pupils who enter the high school find the work of the first year unusually difficult, so that many of them leave school during this period. It has become customary on this account to speak of the "gap" between the eighth grade and the high school, and it is generally admitted that better articulation is necessary there. The efforts to "bridge the gap" have included attempts to modify the work both in first year high school and in the eighth grade. In the eighth grade there has been an effort to concentrate upon essentials of arithmetic on the one hand, and on the other to prepare the pupils for high-school work by introducing the beginnings of algebra into the eighth grade work. There are two tendencies to-day which are likely to lessen the influence of preparation for high school as a modifying force in the eighth grade; first, there is the spirit of independence which characterizes the efforts of each of the larger units of the educational system, as an evidence of which in the elementary school is the belief that their main function is not that of serving as a feeder for the high school; and second, the spirit of responsibility on the part of each unit to take up its work where the previous unit has left off. This latter tendency is much more noticeable among secondary mathematics teachers than formerly.

INFLUENCE OF EUROPEAN CURRICULA.

The object lesson of European schools which provide under one administrative unit educational facilities for children from the age of 9 or 10 to 19 or 20, inclusive, has furnished considerable support for the idea that the essentials of elementary school work should be completed in a shorter time so that pupils may be started upon their secondary education at an earlier age. This has been urged, especially in mathematics, where it means a change particularly in the upper grades. This idea received the support of the committee of fifteen and the committee of ten of the National Education Association.

While four influences have been enumerated, it is clear that each affects the others, so that they are not independent. In particular this last is intimately associated with the preceding one. These two

in a sense are antagonistic to the other two since these place the emphasis upon secondary education, while the others throw emphasis upon the elementary phase of education. The last two are still in favor among the schoolmen especially, as in the past, whereas the former are supported equally by public opinion.

RURAL AND PRIVATE SCHOOLS.

The schools of the rural districts do not feel all of these influences as strongly as do the urban schools. There is a growing effort to adapt the rural schools to the needs of the community, a movement identical with that noted in connection with urban schools. There is strong feeling of the importance of directing attention to the advantages of agricultural life. This influence will increasingly affect the work of the upper grades.

In private nonparochial schools the influences mentioned in paragraphs 7 and 8 are especially potent. The pupils of these schools usually go to the high school and college. In parochial schools there is usually reflected some of the educational policy of the public schools, in addition to the special educational policy of the parochial school. In the eighth grade of parochial schools the special function of these schools reaches its consummation in the specific training of the pupils in such sectarian studies as enable them to enter the church with which the school is connected.

ORGANIZATION OF THE GRADES.

In organization there is nothing particularly distinctive about the seventh and eighth grades. In their relation to the grades below they come under the general school administration which controls the first eight grades. For a discussion of this organization see the report of the committee on general elementary schools. In their relation to the schools above there is some difference. At the close of the eighth grade the pupils are usually given a diploma as a sign of having completed a definite stage of their education. This diploma as a rule entitles the holder to admission to the high school of the same system and usually of any other school system. The diploma is obtained after compliance with the ordinary requirements for promotion. Promotion is determined by the teacher of the class with certain requirements in the way of examination, which will be discussed in a later paragraph. In some cases a special examination is required before a pupil is promoted to the high school.

THE MATHEMATICS CURRICULUM.

There is a lack of uniformity in the curricula in these two grades greater than in the lower grades. Two classes of curricula must be considered: (*a*) Those which do not provide any work in algebra and geometry; (*b*) those which do provide such work.

For those schools which do not provide any algebra in the eighth grade (and in the country at large these schools are in the majority), the mathematics course may be arranged on the topical plan, the spiral plan, or a combination of the two. By a topical plan is meant one in which the subject matter is divided according to mathematical topics; numeration, notation, and the four fundamental operations are discussed in order for integers, then the same topics for fractions in the common form; then for fractions in the decimal form; then percentage; then the applications of percentage to business life. By a spiral plan is meant one in which within a given range of numbers, say one to one hundred, notation, numeration, and certain processes are considered; later the range of numbers, the kind of numbers, the processes, and the difficulty of the applications are extended repeatedly. By a combination of the two plans is meant a course in which, while the spiral plan is adopted in spirit, at various stages of the course certain mathematical ideas are stressed so that thereafter they may be counted on as part of the working knowledge of the class, subject to recall after only slight review.

In the past the topical plan has been especially in vogue; at the present time more progressive courses of study are modeled after the third plan; the second plan has had some support, but has never had very general favor. It is probably true that the topical plan in its extreme form or in a modified form is the common plan in the majority of the schools of the country. By a modified form is meant that typified by a series of texts consisting of two or three books designed to furnish together the necessary material for an eight years' course. Each text usually covers the fundamental operations for integers, fractions, and decimals, with additional topics in each book. One book is to be used for from two to four years.

Usually the mathematics course is guided by such a text. In this event the pupils learn in the first six grades the fundamental processes for integers, fractions, and decimals, and denominate numbers. The work of the seventh and eighth grades consists of the following: A review of this foundation work; special attention to percentage and its most common application, simple interest; other business application of percentage; mensuration of plane and solid figures. The list of applications of percentage includes such topics as bank accounts, partial payments, commercial and bank discount, partnership, insurance, taxes, stocks and bonds, exchange, and interest. In mensuration is included the discussion of the area and the volume of the common plane and solid figures, square, and cube root. Besides these topics, the metric system, longitude and time, ratio, and proportion are commonly included in the texts. On account of the influences noted in former paragraphs, the tendency in the better schools of

the country is to work for thoroughness and utility by omitting such topics as partial payments, partnership, exchange, bank discount on interest-bearing notes, the more difficult work in stocks and bonds, longitude and time, some portions of the mensuration of solids, cube root, and the metric system. In short, there is great freedom in omitting such topics as are not in harmony with modern life or as have been found unnecessarily confusing for the pupils. The time thus gained is devoted to the remaining topics and to additional drill on the essentials.

The problem material is contained as a rule in the book which each pupil possesses. These texts are written for the country at large, not to suit local conditions. The problems are designed to illustrate the particular mathematical idea or application under discussion; they are usually miscellaneous in character. The influences noted in former paragraphs are responsible for a new attitude toward the problem material. An effort is now made to have one set of ideas running through a particular set of problems. For example, the arithmetical idea being discussed may be percentage; instead of giving as problems a miscellaneous lot of applications of the percentage idea, the problems may all be based upon information about the population of the country for a period of years. In other words, the problem material possesses a certain unity, as a result of which information is conveyed to the pupils on the subject from which the material is drawn. A second characteristic of the problems now coming into use is that they are selected so as to give the children some insight into industrial, business, and social conditions of the city, the State, and the country. Thus there are groups of problems about the railroad, the mining, the agricultural, the manufacturing interests; about the population, area, and the wealth of the city, the State, the Nation; problems drawn from the local newspaper; problems involving the local tax rate, local real estate values, local interest, discount and commission rates, the stocks and bonds of local corporations. This material the teachers must place before the pupils either orally or on the blackboard, although some school boards are issuing pamphlets containing these supplementary problems for use in their own schools. This is a serious attempt to socialize arithmetic. It makes the course consist less of figuring and more of discussion. Under banking for example, the nature, function, and conduct of a bank is discussed; under stocks and bonds, effort is made to show again the kinds, purpose, value, and manner of sale of bonds and stocks; under taxes, the various purposes for which taxes are collected and the sources from which they are obtained. The content of the problems is considered quite as important as the solution of the problems.

COURSES CONTAINING ALGEBRA.

The other type of course in mathematics includes some work in algebra, and sometimes in geometry. These courses vary even more than those discussed previously, because the work is relatively new. In some places the arithmetic work of these two grades is curtailed and condensed so as to be completed entirely by the end of the seventh grade or by the end of the first half of the eighth grade, the latter being the more common plan; the remaining time is then devoted to algebra. In other places, work in algebra is introduced into the seventh and eighth or the eighth grade without dropping the arithmetic; there is in this case little effort to bring about any vital connection between the two subjects; the time is merely distributed so as to cover the two subjects. Again, in a few places, an effort is made to introduce the algebra into the arithmetic course in a natural way.

When algebra is taught in either of the first two ways, the work is guided by a text. This text is in some cases a brief elementary algebra written for the eighth grade classes, or consists of a few chapters added on to the arithmetic text. As a rule, the sequence of topics covered is that of the usual high-school text. When the algebra is introduced in the third form, it is presented through the regular arithmetic text, where it is introduced as opportunity permits. For example, preceding the work in percentage, the first notions of literal numbers are introduced; then the percentage law is expressed as a formula, $p = br$. Thereafter this formula is used in the solution of problems, for example, such as require the determination of the rate when the base and the percentage are given. Later this literal arithmetic may be extended, although it is quite clear that no very extended amount of algebra can be introduced in this natural manner.

So far an attempt has been made to present briefly the various forms in which algebra is taught in the grades. No definite statistics are available to form the basis of a definite statement as to the prevalence of the teaching of algebra. One investigation showed that about 30 to 35 per cent of the schools of a certain class taught algebra and these were schools of the larger cities; taking the country at large, it is unlikely that more than this percentage of them teach the subject.

PURPOSE OF ALGEBRA IN THE GRADES.

Algebra was introduced into and is retained in the grades for a number of reasons. Some hoped thereby to interest some pupils in the subject to such an extent that they would be led to enter high school to complete it; others hoped that thereby the pupils who did

enter high school would have a more successful time in their mathematics work—in other words, it was a device for "bridging the gap." With others, there was the desire to "abridge and enrich" the arithmetic course; after abridging it, they felt that the natural means for maintaining the mathematical element in the curriculum was to introduce the next higher mathematical subject, which happened to be algebra. It was hoped by some that the use of the literal number and of the equation would eliminate some of the difficulties the pupils experienced in analyzing and solving some of the arithmetic problems in the eighth grade course. Those interested in the mathematical element in education desired to carry to another stage of their logical generalization certain arithmetical ideas.

Two tendencies in regard to the position of algebra are to be noted. With the public at large, the subject has never appeared as one of any "practical" value. With the current tendency to exalt in the schools those subjects which appear to have "practical" value is coming a feeling that algebra does not have a place in the eighth grade, especially in the form in which it has been taught. One of the largest cities of the country has recently entirely dropped it from the course, replacing it by the study of certain phases of local history and local business and industrial conditions. Another city, somewhat smaller in size but recognized as progressive in educational efforts has introduced about the same idea into its course, although there the next tendency to be mentioned has been recognized. These efforts, entirely independent, are typical of the feeling in some of our school systems.

The other tendency is toward an extension of the third plan of teaching algebra. There is in this country a growing belief that in the past too sharp lines of demarkation have existed between arithmetic, algebra, and geometry and there are efforts being made to bring the three into closer relation. In the elementary school, as early as the sixth grade, certainly no later than the seventh, certain elements of generalized arithmetic should be introduced and should be carried along with the arithmetic until the pupils become familiar with literal notation and the equation. This work is not properly called algebra as the number field would not necessarily be extended to include negative as well as positive numbers. This form of literal arithmetic presents all that is practical of algebra for ordinary purposes. So much of it is decidedly of utilitarian value since the mechanics of to-day who wish to read trade journals need to be familiar with it. This form of literal arithmetic has in its favor most if not all that can be said for algebra as an eighth-grade subject.

It seems probable at the present time that these two tendencies will become more pronounced, i. e., there is almost certain to be more dissatisfaction with the extreme form of algebra brought down from

the high school, and as teachers are trained for the work, there is to be, it is hoped, greater interest in the form of generalized arithmetic outlined.

GEOMETRY IN THE GRADES.

The mensuration of certain geometric figures has always been included in American arithmetics. In recent years, as has been said, there has been a decrease in the extent of this work. It was formerly the practice to have this work consist mainly of definitions, formulas, and problems. This has been changed by giving experimental and intuitive verifications of the formulas. It has been uncommon to have any other form of geometry, except in isolated places where some form of constructional, inventional, or concrete geometry has been introduced. The motive has been to teach certain elementary ideas of geometric forms, to train the hand in the use of customary drawing tools, to train the eye in its judgment of geometric forms and relations, and to train the mind in such general functions as observation and generalization as pertaining to geometric data. This work has been attempted usually under the guidance of a pamphlet prepared for some specific school by those interested in such work.

The movement has not spread much. Possibly one reason for the slowness of the introduction of this work has been the custom of including in the art course usually given in American schools some "mechanical" drawing—constructions with the straightedge and compasses. This custom has been unfortunate, at least from the mathematical standpoint, as, in the drawing course, the emphasis has been upon the results; the possible cultivation of desirable habits of geometric study, and the possible training of the powers of observation and generalization have been largely neglected. A further reason for the lack of such is the scarcity of teachers acquainted with and prepared to teach geometry inductively. The majority have studied geometry only in the Euclidean form.

EXAMINATIONS.

Examinations given in the schools are of three kinds: (*a*) Those given by the teachers themselves; (*b*) those given by the supervisory officers of the schools; (*c*) those given by a school to determine the qualifications of pupils who wish to enter the school.

The examinations given by the teachers are of two kinds: The ordinary written recitation and the stated examinations which may be required by the school regulations. The first should not be called examinations in one sense of the word, since they cover usually only a short interval of previous instruction and are given as a means of affording the same sort of drill for all of the class or as a means of detecting weaknesses as a basis for further teaching. Such written

lessons are left entirely to the teacher, although it is advised that they be given frequently. Such "examinations" are in every respect desirable.

The other form of examination given by teachers is more formidable. Most school systems provide that the standing of a pupil in a class shall be determined in part by examinations given at stated times. The examinations may occur monthly, quarterly, semiannually, or annually. They count from one-half down in determining a pupil's standing. When the questions are prepared by the teacher, the questions and the papers are at the disposal of the principal or other supervisors. The teacher has an opportunity to adapt the examination to the capabilities of the class, and can allow for the individuality of the pupils. Under this plan a sympathetic influence is likely to pervade the examination. As a means of administration this form of examination depends for its success upon the teachers and the opportunity of inspection. As an educational practice it is to be commended as compared with those examinations which are not prepared with the same sympathetic recognition of the pupils' interests, which the teachers are likely to display.

As a rule uniform tests are given in most school systems for administrative purposes. The questions are prepared either by the superintendent or by a committee of principals or teachers—usually principals—working under the direction of the superintendent. In few cases do the teachers have any choice concerning the questions which they submit to their classes. The teachers grade the papers and then submit them to the principals. These examinations are given to set standards of work, to interpret the course of study, to promote uniformity throughout the system, to bring out the weak points in the teaching, and to point out conditions in the school. In some cases it is urged that these examinations train the pupils to prepare their thoughts on a subject in good order in a limited time. In general the interests of the pupils are served only indirectly by these examinations as the emphasis is upon the administrative advantages. The results are sometimes used in determining a pupil's fitness for promotion, although it is seldom that failure in these examinations is allowed to retard a pupil's progress.

These examinations are an effective administrative device. Their success depends upon the experience, the wisdom, and the ideals of the supervisory staff. As a rule, these examinations do not meet with favor among the teachers. There is a feeling that the test is one of themselves rather than of their pupils. From what has been said this appears to be true, although it depends upon the purposes of the supervisors. The better teachers recognize the advantages to be gained, and, having the proper professional spirit, they are willing to have their work compared with that of their colleagues in other

schools. The examinations are opposed also because of their effect upon the pupils. It is contended that the pupils are subjected to a severe nervous strain so that they do not do themselves or their teachers justice. This is possibly true. The evil results, however, may be minimized under wise supervision. If the teachers adequately prepare the pupils for the tests by reviews, and if the tests are adapted to the possible ability of the pupils, the evil results mentioned are not necessary. In the larger cities the difficulty of conducting these examinations is great, owing to the wide diversity in the population in various parts of the city. Another objection raised to such examinations is that thereby the teachers are hampered in their work, with the result that there is little progress from year to year. This again may or may not be true, dependent upon the supervision. As a rule, the supervisors of the schools will see that the tests promote rather than retard progress.

Another form of examination proposed at the present time is that designated as the "standardized" test. For a complete discussion of the nature of these tests, see the report by Dr. C. W. Stone, in the report of subcommittee No. 2 of this general committee. As to current practice, it is safe to say that little work of this nature is done in the schools. As to tendencies, there is little evidence upon which to base a statement one way or another. It would seem well for those in charge of the general tests just discussed to introduce into them such of the elements of these standardized tests as seem applicable to their needs and purposes.

In the eighth grade tests are sometimes given to determine the fitness of pupils for promotion to high school. This is not a common practice, however.

METHODS OF INSTRUCTION.

It is possible to speak only in a general way of methods of instruction as these vary with schools and with teachers. Much that can be said on this topic for the two upper grades would apply equally well for the other grades.

CLASS INSTRUCTION.

In all of the schools instruction is given to groups of pupils varying in number from 5 to 30 or more—groups called classes. The average size of classes is probably in the neighborhood of 20 and a strong effort is being made in all cities to cut down larger classes to this number. This form of instruction is called class instruction. There have been numerous attempts to modify this form of instruction by various forms of individual instruction in order to meet in a better way the needs of weak pupils. In some schools special teachers are

employed to take charge of any pupils from any class who are backward in their work. As a rule, however, the regular class teacher does this herself either before or after regular school hours. An effort is made of course to organize the classes so that all pupils in the class will have as uniform ability as possible.

RECITATION AND STUDY TIME.

In most schools the arithmetic class meets daily for from 20 to 30 minutes. This time is known as recitation time. Besides this the pupils of the class usually have in school another period of equal length for the study of arithmetic. This makes the total time for arithmetic vary from 200 to 300 minutes per week. In most schools the children do no studying on arithmetic at home; in the upper grades they probably do, although the tendency to-day is to relieve pupils of any home preparation in mathematics in the elementary schools. Each pupil possesses a book. This book is not merely a collection of problems; it is usually a text providing the necessary theory and such explanation as seemed wise to the author. From the terminology used to denote the two periods, it is obvious that at some time the pupils were expected to prepare themselves in the "study" period on certain assignments in the text upon which they later recited in the "recitation" period. This was especially true in the upper grades, and unfortunately is probably true in many classrooms today. In the majority of schools this condition has undoubtedly changed; the recitation period should properly be called the "teaching" period and the study period might better be called the "work" period. The class time was formerly given over often to indiscriminate recitation on the solutions which the pupils had performed outside of class; it is now given over either to carefully planned drill or to instruction by the teacher on some new topic. The study time is used to supplement the class time.

DIVISION OF THE CLASS TIME.

One of the characteristic features of the teaching period is the mental work. It is common practice to direct the teachers to devote from one-third to one-half of the class time to oral-mental work. It is oral in the sense that the teacher gives the directions orally; it is mental in the sense that the pupils perform the necessary computations without use of pencil and paper. The responses of the pupils are given either orally or in writing. This work is designed either to maintain efficiency through wise drilling on topics previously taught or to lead up to and teach some new topic. This must be considered a feature of current teaching method, since formerly much of the class

time was given over to "recitation" by the pupils on work which they placed upon the blackboard.

There are occasions when it is worth while to have the whole class solve problems in writing; this is especially true in the upper grades where the conditions of the problems become more confusing. As a rule, however, written work is done outside of class. In the past pupils have been required to present more or less elaborate analyses of the solutions for their problems. The desire was to have the pupil set forth in detail the process of thinking by which the solution was accomplished. The advantage of such solutions to the teacher is obvious; it is an equally obvious fact that such formality is entirely foreign to the natural, rational mode of presentation which an adult would use in ordinary life. Such a requirement is subject to criticism also because it confuses the pupils. The tendency now is to ask the pupils to give a clear presentation in good form; in the upper grades, they are encouraged to use any "short cuts" of which they know, and to do mentally as much of the computation as they can. Another reform which has been accomplished is the discontinuance of elaborate forms of ruling on the papers, a practice which has been altogether too prevalent in the past.

RAPIDITY AND ACCURACY.

The teachers of the upper two grades seek to develop in their pupils skill in computing which will enable them to perform ordinary calculations with rapidity and accuracy. In the lower grades the pupils are taught the processes and the number facts; in the upper grades the emphasis is properly turned in this other direction.

The mental work has this as its aim. The pupils are frequently given problems to solve in a limited amount of time; they are encouraged to compete with one another by being invited to rise when they have solved the problems; they are given honorable notice in various ways for achievement in these respects.

CONCRETE METHODS.

On account of the influences mentioned in former paragraphs, the teachers endeavor to make the instruction as concrete as they can without going to the extreme of objectifying relations which are obvious. Thus in the discussion of commercial forms, actual samples are exhibited; checks of some real or imaginary bank are drawn in class; account books are kept by the pupils and are balanced monthly, each child filling in records of real transactions when possible; interest-bearing and noninterest-bearing notes are shown; banks are visited or conducted in the schoolroom; the class resolves itself into a stockbroker's office, one member acting as the broker and the

remaining members of the class acting as buyers or sellers of stocks or bonds. In mensuration, actual measurements are used when possible; problems are sought in the manual training department; formulas are obtained by experimental methods instead of being taken on faith and then used. The teachers endeavor to give to their classes clear impressions of a few things rather than superficial word knowledge of many things.

DEPARTMENTAL TEACHING.

One especially characteristic feature of the seventh and eighth grade work is the practice of having the mathematics, and some of the other subjects, taught by teachers who as a rule teach no other subjects. In the lower grades this is not common. This is known as departmental instruction. It is quite common in the larger school systems and is being extended as rapidly as circumstances permit. The teachers in charge of the departmental work are usually the more experienced teachers of the system and as a rule have had special training for their work. In many cases college graduates are obtained for these positions, whereas usually the teachers in the elementary schools do not have collegiate training.

Departmental instruction is favored because of the obvious advantage of having for this upper grade work teachers who are specially interested in and qualified to teach their subject. Departmental instruction has the disadvantage of placing the training of young children in the hands of each of several teachers, working independently, with the result that often the emphasis is placed upon teaching the subject rather than upon the education of the child. This difficulty is obviated by wise supervision, under which there will be cooperation between the various departmental teachers in any one school.

INDUCTION AND DEDUCTION.

It is very likely that no conscious thought is given to these two types of teaching methods by the majority of teachers. At the same time it may be said that the instruction is either by inductive methods or by rule followed by practice. The habit of proceeding from "the known to the unknown," from "the simple to the complex," is so characteristic of the teachers that they approach new topics whenever it is possible by inductive means. For example, the nature and the meaning of "paying interest" for the use of money may be approached through the acquaintance the pupils have with the practice of paying "rent" for the use of a house; the nature and meaning of shares of stock may be approached through the pupils' experience in contributing their share toward the expense of a picnic. On the

other hand, when it comes to a process such as finding the square root of a number, the books as a rule develop the process in the usual way with the aid of the formula for the square of a binomial; but the teachers are commonly advised to simply illustrate the process and then fix it in the minds of the pupils by adequate drill.

THE PAROCHIAL SCHOOLS.

This investigation covers the field embraced by Roman Catholic parochial schools, which number in the United States 4,845, having a total registration of 1,237,250 pupils. These schools outnumber other schools maintained by any single church organization, and for this reason they are selected for this investigation. In general they do not charge any tuition, unless to pupils not residents of the parish. They are in most cases supported by the Roman Catholic Church from a fund voluntarily contributed by members of the church. In many instances, however, where such a fund does not exist tuition is charged at 50 cents a month for the lower grades and $1 a month or more in the higher grades.

The committee finds none of them to be endowed in any way or to receive any State or municipal support.

The Roman Catholic Church divides the United States into 70 dioceses, in each of which a board of supervisors has charge of the educational work in the parish schools. It is the business of the diocesan board to make out the course of study for the various schools, select the books to be used, and to pass on any matters pertaining to the work of the schools.

The teachers, being generally members of religious orders, are appointed by the superior of the order to which they belong, as recommended by a committee acquainted with the attainments of the candidates. The different religious orders have a community inspector of schools who reports to the superior on the efficiency of the teaching staff. The following syllabus of mathematics as taught in the seventh and eighth grades is selected as representative of the better class of schools and is from the Philadelphia diocese.

ARITHMETIC—SEVENTH GRADE.

(1) Drill exercises for accuracy and speed. (2) Miscellaneous problems involving fractions. (3) Multiplication, division, and miscellaneous problems on decimals. (4) Denominate amounts: (a) Measurements, extending to circle and cylinder; (b) compound amounts in use; (c) longitude and time. (5) Percentage: (a) Review and extension of sixth year's work; (b) bank discount. (6) Ratio and proportion: (a) Ratio; (b) simple proportion; (c) proportional parts.

ARITHMETIC—EIGHTH GRADE.

(1) Common and decimal fractions; underlying principles considered and applied. (2) Denominate amounts. (3) Percentage: (a) Review and extension

of seventh year work; (*b*) stocks and bonds; (*c*) compound interest; (*d*) drafts and exchange; (*e*) partial payments. (4) Proportion and partnership. (5) Powers, roots, and mensuration. (6) Review of practical measurements: (*a*) Plane surfaces; (*b*) triangles, quadrilaterals, regular pyramids, right cylinders, right cones, frustums of regular pyramids, and cones, spheres; (*c*) similar figures.

ALGEBRA—SEVENTH GRADE.

(1) Explanation of algebraic terms to be employed and illustrations of their uses. (2) Literal expressions; their numerical value obtained. (3) Comparison of arithmetical and algebraic solutions of simple problems, and solution by algebra of many such problems. (4) Positive and negative quantities explained and practical problems employing their uses considered. (5) Addition and subtraction. (6) Parentheses. (7) Multiplication and division.

ALGEBRA—EIGHTH GRADE.

(1) Review of addition, parentheses, multiplication, and division. (2) Composition and factoring. (3) Highest common divisor and lowest common multiple. (4) Fractions. (5) Simple equations.

The average age of pupils entering the seventh grade is between 12 and 13 years, and at leaving the eighth grade about 14 years. In about half the schools reporting, promotion from grade to grade is determined by the teacher. In the remaining schools promotions are recommended by the teacher and determined by the principal. All schools average the class work and examination in determining promotion, usually each counting 50 per cent, but a few giving two-thirds credit for class work. In the case of seventh and eighth grade examinations, some schools give the questions as made out by the city superintendent of public schools, and papers are marked by public school authorities for eighth-grade examinations. Where Catholic high schools exist the principal and teachers make out the questions, while the examination is conducted by the teacher and the papers are inspected by higher authorities. About one-fourth of the schools permit of a choice of ten questions out of eleven, the others allowing no choice.

Regarding promotion after failure to pass in a mathematical subject, a wide diversity of practice exists. The question asked in this respect was, "To what extent does failure in a mathematical subject retard a pupil's progress?" Some answers received were as follows: "As a rule, a mark below 60 per cent in any mathematical class requires a repetition of the class"; "Considerable deficiency influences a pupil's promotion"; "Not kept longer than two years in a class"; "Mathematics is one of three studies on which promotion is based"; "Conditional promotion is given, but is withdrawn if progress is not made"; "Must cover program again"; "Must stay in class"; "Those notably deficient are not promoted"; "Debars pupil from promotion"; "Passes to next year's grade if only failure is in mathematics."

Promotions are made annually or semiannually about equally. In regard to the object influencing the teaching of mathematics, preparation for high school stands first, the demands of the business world second, and about 20 per cent report discipline and culture as the predominating influence.

Oral work in class, which is given in all schools, varies in extent from 12 minutes to half the recitation period daily, and nearly all schools report a preliminary discussion of newly assigned topics until a sufficient knowledge to proceed has been obtained.

The size of class varies in different localities from 10 to 65, the average class for the seventh grade being 34, and for eighth grade, 27. The usual time spent in recitation per week is 200 minutes with 150 minutes per week given to study in school. There is also some home work given that requires about 30 minutes nightly.

In the eighth grade, the time given to recitation and study are about as shown for seventh grade, and some algebra is often introduced.

Reforms contemplated are, "more oral work," "to apply the work to problems of personal interest to the pupil," "to eliminate the nonessentials and emphasize the essentials," "better work in oral arithmetic," "more drills for thoroughness."

SUBCOMMITTEE 6. PREPARATION OF TEACHERS FOR GRADES 7 AND 8.

In preparing this report the committee attempted to obtain information from both public and private elementary schools in accordance with plan suggested by the American commissioners. Practically no response was received from private schools to which blanks were sent, and hence the report is limited to public elementary schools.

I. PRESENT STATE OF ORGANIZATION.

TEACHERS.

In investigating the present state of organization, the tabulation of elementary schools given by the United States Commissioner of Education in his report for 1908, volume 2, pages 502–517; 596–611, was used. In these tables are listed 531 schools in cities of over 8,000 population, and 670 schools in cities of 4,000 to 8,000 population. The first group is known in the report as Class A and the second as Class B.

Five hundred cities, 250 from each class, were selected for the purposes of the report, each member of the committee selecting 100 from the section assigned to him, 50 from each class. Only graded

schools were chosen, and each State was considered in proportion to the number of its school systems.

In response to a questionnaire sent to the 500 cities, 238 replies were received as follows:

NUMBER OF CITIES REPORTING.

	Class A.	Class B.
North Atlantic	29	19
South Atlantic	13	11
South Central	13	11
North Central	64	54
Western	13	11

The following tables summarize the information received from the cities:

TABLE 1.—*Teachers.*

	Number in seventh and eighth grades.				Preference of cities as to sex of teachers for seventh and eighth grades.					
	A		B		A			B		
	Men.	Women.	Men.	Women.	Men.	Women.	No preference.	Men.	Women.	No preference.
North Atlantic	679	2,642	11	121	11	3	0	7	6	3
South Atlantic	41	164	16	37	5	3	1	5	3	....
South Central	40	155	10	36	9	1	2	5	4	....
North Central	102	1,602	55	294	28	17	9	31	7	8
Western	35	354	20	69	8	1	1	5	1	1
Total	987	4,917	112	559	61	25	19	53	21	12

TABLE 2.—*Preparation of present teaching force in grades 7 and 8.*

	Actual.			Preference by cities.		
	College.	Normal.	High school.	College.	Normal.	High school.
North Atlantic:						
A	895	316	953	13	15	....
B	9	90	126	3	16	....
South Atlantic:						
A	48	41	110	11	7	....
B	18	19	41	1	9	....
South Central:						
A	62	52	139	8	3	2
B	19	9	31	4	6	2
North Central:						
A	115	958	1,727	21	38	....
B	50	111	311	19	31	1
Western:						
A	13	121	137	5	7	....
B	21	52	55	4	4	2
Combined:						
A	1,133	1,488	3,066	58	70	2
B	117	281	564	31	66	5
Total	1,250	1,764	3,630	89	136	7

TABLE 3.—*Present requirements of cities as to preparation for teachers of grades 7 and 8.*

	Academic.				Professional training.				Certificate.					
	College.	Normal.	High school.	None.	Two years.	One year.	Experience.	None.	State.	First grade.	Second grade.	City.	None.	Normal school diploma.
North Atlantic:														
A	3	12	11	5	15	1	7	4	5	....	1	5	14	2
B	....	5	8	4	10	....	3	5	4	....	....	....	10	2
South Atlantic:														
A	2	5	4	4	3	....	3	6	....	7	....	2	1	2
B	....	5	2	....	2	....	4	....	1	6	....	....	....	1
South Central:														
A	2	4	5	1	5	1	....	5	4	2	1	4	1	1
B	3	2	3	2	1	....	3	4	1	3	2	1	2	......
North Central:														
A	3	16	39	11	27	5	14	13	4	21	13	13	7	12
B	1	11	22	16	18	6	12	13	6	18	14	2	6	10
Western:														
A	....	4	9	....	10	....	1	....	1	3	6	....	2	......
B	1	3	4	....	7	....	1	2	2	2	4	....	1	......
Combined:														
A	10	41	68	21	60	7	25	28	14	33	21	24	25	17
B	5	26	39	22	38	6	23	24	14	29	20	3	19	13
Total	15	67	107	43	98	13	48	52	28	62	41	27	44	30

In New York City the preparation required of men teachers in grades seven and eight is: (1) College course and one to three years' experience; or (2) a normal school course and three years' experience. In addition there is required a promotion license based on, (1) three years' experience; (2) 60 hours' professional course; (3) examination in arithmetic, algebra, and geometry. For women teachers the requirement is (1) normal school course and one years' experience; (2) New York State life certificate and five years' experience; or (3) college course and one years' experience.

In two cities of Class A and three of Class B, men are preferred for the eighth grade and women for the seventh. In New York City men are preferred for teaching boys and women for teaching girls. In Auburn, N. Y., and seven other places reporting, the preference is for an equal division of men and women teachers in the grades in question. There is a general opinion that on the present salary basis women teachers are preferable to the men who are willing to work at present rates of compensation.

COURSE OF STUDY.

The course of study in mathematics for the seventh and eighth grades of the public schools throughout the United States is remarkable for uniformity. In the seventh grade the usual topics are denominate numbers and percentage; in the eighth grade, applications of percentage—discount, insurance, notes, stocks and bonds, etc. These topics are supplemented by review or advance work as time allows. For example, in a typical school we find in the seventh

grade in addition to the topics named above, analysis of problems, ratio, fractions, decimals; in the eighth grade, proportion, powers and roots, longitude and time.

The excellent course of study in mathematics for the public schools of Augusta, Me., says: "Up to this time (seventh grade) the pupil has been, to a large extent, a mere reckoner; he should now become something of an arithmetician. The mathematical mind was a late development in the race, and its growth in the child should not be forced. It is believed, however, that this grade marks the approach of the time when the pupil should pass from mere control and manipulation to understanding and investigation. He may now be asked to define arithmetic, or at least attempt a definition, and should appreciate some of the difficulties of its development.

"No discrimination need be made between algebra and arithmetic, nor between algebraic and arithmetical solutions. Pupils should not feel abashed in the presence of x, but should regard it as a good friend, employ it habitually, and if possible, unconsciously, to represent unknown quantities, and solve the equations containing it entirely as a matter of course."—Course of study in mathematics for the elementary schools, Augusta, Me., 1909

In this connection it is interesting to note that of the 238 cities reporting to the committee 11 of Class A and 7 of Class B teach algebra in the seventh grade, while 47 of Class A and 26 of Class B teach it in the eighth grade. Concrete geometry is taught in the seventh grade of 8 Class A schools and of 3 Class B schools; it is taught in the eighth grade of 10 Class A schools and of 8 Class B schools.

In New York City the consensus of opinion of the large majority of principals is that the inventional geometry and algebra of the seventh and eighth grades should be entirely omitted; that these subjects should be relegated to the high school, and that the time gained should be given to pure arithmetic.

In Chicago the board of education has recently voted to drop algebra and geometry from the seventh and eighth grade courses.

A very few schools reported such unusual features as the correlation of arithmetic, algebra, and geometry, and the segregation of pupils. The department plan is used by 103 schools.

In Class A 99 schools reported the present course satisfactory, while 30 reported the opposite. In Class B 58 schools reported course satisfactory and 39 reported unsatisfactory. As causes of dissatisfaction 83 gave "poor teachers"; 49, "course of study"; 12, "large classes"; 10, "lack of time"; 13, "poor pupils"; 12, "textbooks."

In response to a question relative to the attempts to "humanize mathematics," 68 report "good"; 30, "none"; 27, "doubtful." The effect of manual training is considered "good" by 104, "none" by 9, and "doubtful" by 35.

Among recent improvements, 28 report "none"; 64, "course of study"; 51, "teachers"; 32, "texts"; 3, "salaries"; 9, "department teaching."

II. THE TRAINING OF SEVENTH AND EIGHTH GRADE TEACHERS OF MATHEMATICS.

1. CITY TRAINING SCHOOLS.

Of 132 Class A cities included in this investigation, 29 report city training schools with an attendance of 127 men and 3,222 women. Of this number, 113 men and 2,182 women are reported from the three New York City training schools.

The plan of organization varies from the St. Louis five-year course for high-school graduates to that of a department of the high school. Local conditions govern in this matter. Theory and practice are about evenly balanced in all cases. Academic instruction varies from the first two years of a university course, as in Cincinnati, to none at all in 11 schools reporting.

Fifteen of these training schools give opportunity for practice teaching of mathematics in grades 7 and 8. The longest practice period reported is that of Cambridge, Mass. In that school one year of practice teaching is offered and cadet teachers are paid $200 to $250 per year.

Seven city training schools offer courses in the history and teaching of mathematics; one mentions as a special feature a mathematical library.

In response to a question as to the conditions which a rational preparation of teachers for the grades in question should fulfill, replies were received as follows: Massachusetts—(*a*) normal or college diploma, (*b*) summer school course at least every third year, (*c*) membership in local mathematics club, (*d*) membership in New England Mathematical Association; Virginia—(*a*) high-school course, (*b*) good normal course, with practice work; Georgia—(*a*) teaching knowledge of subject, (*b*) practice course, (*c*) rational course in pedagogy; Indiana—inexperienced teachers should not be allowed in seventh or eighth grade work; Iowa—good knowledge of arithmetic, algebra, and geometry; Missouri—more attention to subject matter, but method not neglected. The remaining 23 cities offer no suggestions on the question.

2. UNIVERSITY AND COLLEGE DEPARTMENTS OF EDUCATION.

Sixty university and college departments of education reported as follows: North Atlantic, 7; South Atlantic, 10; South Central, 6; North Central, 32; Western, 5.

Twenty of these schools offer opportunity for general practice teaching in mathematics and 15 give practice in grades 7 and 8 of their training school. Twenty-two offer course in the teaching of mathematics and 16 in the history of the subject. Research work is

done in 13 schools. In the rational preparation of teachers, 35 believe that emphasis should be laid upon subject matter, while 21 would stress method.

In all of these schools students preparing to teach have access to the regular college and university courses in mathematics.

3. STATE NORMAL SCHOOLS.

Reports were received from 88 State normal schools, as follows: North Atlantic, 17; South Atlantic, 5; South Central, 8; North Central, 36; Western, 14.

Seventy of these schools require general practice teaching in mathematics, while 66 require it in grades 7 and 8 of their training schools. Courses in the teaching of mathematics are offered by 52, and courses in the history of mathematics by 14 schools, while 5 give opportunity for research work.

The mathematics course in normal schools varies greatly. The minimum is a high-school course. About one-fourth offer college courses in mathematics. Nearly all of this group are in the North Central and Western States.

4. PRIVATE NORMAL SCHOOLS.

Forty-two private normal schools sent in reports. They were distributed as follows: North Atlantic, 3; South Atlantic, 19; South Central, 8; North Central, 11; Western, 1.

Of this number 28 have practice schools, and 24 give special training in teaching seventh and eighth grade mathematics.

Courses in the teaching of mathematics are offered by 20; in the history of the subject by 4. Two of them give opportunity for research.

In connection with both public and private normal schools, it is a matter of special interest that, contrary to the usual opinion, the majority lay emphasis upon subject matter of mathematics in teacher preparation rather than upon method. Sixty-one would emphasize subject matter, while 22 would lay the emphasis upon method.

5. THE FORWARD MOVEMENT IN NEW YORK.

(*a*) In New York City in April 1911, the board of education took action requiring all teachers not holders of a grade "A" license to pass an examination before promotion to positions in seventh and eight grades. For the improvement of the teaching of mathematics in grades 7 and 8 the following steps have been taken:

1. Special attention to the subject by many of the district superintendents in principals' conferences and in holding tests for different grades uniform throughout their respective districts.

2. The issuance of an important special circular by the city superintendent indicating the main errors committed in teaching mathematics in the city schools and pointing the way to remedy those errors.

3. The establishment of courses of instruction in the history of mathematics and in methods of teaching elementary mathematics. Some of these courses have been given under the independent auspices of teachers' associations, others have been given in cooperation with local universities, and others have been given independently by local universities.

4. The local teachers' and principals' associations of the Borough of Brooklyn have jointly prepared a carefully devised general teaching plan in mathematics for the purpose of securing desirable uniformity in the sequence of topics in the schools of the borough. This plan is widely used.

(*b*) In New York State revisions of the graded-school course are in progress, which will very materially affect the work of the next five years. The following extract from the 1910 report of Commissioner Draper indicates the nature of the revisions:

A syllabus covering the first six grades is submitted. * * * It has not been possible nor was it expected that all of the present work of the seventh and eighth grades could be put in the six years' course. * * * The intermediate course covering such grades to round out the elementary course has been prepared. It is in this two years' (seventh and eighth years) course that the elementary work will begin to differentiate. The regular course will lead to the present high-school course. It includes arithmetic, history, English, physiology, modern languages, and other subjects which have previously been considered in academic work. * * * This two years' course also includes work specially adapted to prepare pupils for commercial and industrial courses in high schools and in trade schools.

6. GENERAL OPINIONS CONCERNING THE RATIONAL PREPARATION OF TEACHERS FOR GRADES 7 AND 8.

Some of the general opinions concerning rational preparation of teachers are noted here. They are fairly representative of the whole group.

From a Pennsylvania normal school:

A more complete appreciation of the subject matter, emphasizing its philosophy and the need of absolute accuracy of statement and of result. In addition to this must come an appreciation of the mental activity to which the various phases of the mathematics make their appeal, why this is so, and the means of making this appeal most effectively—this I think of as the pedagogy of mathematics.

From Commissioner Draper's 1910 (New York State) report:

The normal schools are charged with the special duty of training teachers for the elementary schools. * * * The elementary work of the seventh and eighth grades is to be so radically changed under the new syllabus that special provision must be made for training teachers for these grades so that more men will enter this field of teaching.

From a New York State normal school:

(1) It (rational preparation) should give a thorough knowledge of the subject matter of arithmetic; (2) it should emphasize the development of the

subject from the beginning, and the methods of teaching the subject; (3) it should include a discussion of modern movements and modern tendencies, and the consideration of the different classes of material with which the teacher should be provided. * * * The average normal pupil's knowledge of algebra is too limited to enable him to follow any discussion of methods of developing topics in algebra.

A new Jersey State normal school:

Students who come to us from high schools which give the best preparation in mathematics are the best students in methods of teaching arithmetic. Therefore a round course of mathematics before methods of teaching.

Southwest Texas Normal School:

When the entrance requirements are low, nearly if not all the time should be given to the acquisition of subject matter. As entrance requirements are raised, more time should be given to the professional work of the subject.

University of Texas School of Education:

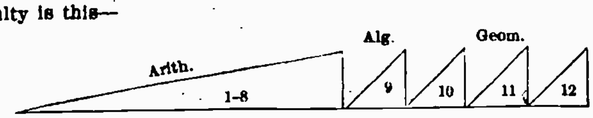
The training of teachers of mathematics for grades 7 and 8 should include these features: (*a*) Scholarship, including solid geometry, trigonometry, advanced algebra, and some analytics; (*b*) a course in general method; (*c*) a course in psychology, including developmental psychology; (*d*) a course in the history of education; (*e*) a course in practice teaching.

Washington State Normal School:

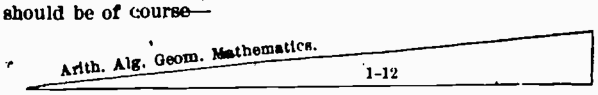
I am inclined to think that more attention should be given to subject matter, but not any less to the method. We have so many "isms" that mathematics is not receiving its just share of time. Most students coming from accredited high schools are weak in the subject matter of arithmetic.

Washington State University department of education:

The great weaknesses of mathematics in the grammar grades are two: (1) The absurd American plan of grinding out complex and almost impossible arithmetic before taking up enlightening and comparatively simple elements of algebra and (plane?) geometry. This is an intolerable fault. Our actual curve of difficulty is this—



when it should be of course—



(2) Mathematics should be more concrete in the grammar school (and I think even in the high school), and (*a*) more clearly related to the boy's interest, (*b*) more closely correlated with other subjects.

Montana State Normal College:

We find that it is very difficult in the time we have here to make students satisfactorily reliable in arithmetical subject matter. High-school graduates seem to have lost completely all the knowledge of arithmetic presumed to have

been acquired below the high school, except the little which they use in higher mathematical subjects, and even in that little they are rarely accurate.

Iowa State Teachers' College, President H. H. Seerley:

Mathematics has a proper place from the fourth grade through the twelfth grade, provided it is adapted to the age and development of the pupil. All the arithmetic necessary to be known can be taught in less than a year if given at the proper time of the pupil's development. There is a great waste of time now, as children are treated as if they need adult knowledge. A suitable book on mathematics should be written for the grades below the high school.

CONCLUSION.

The committee has attempted to set forth the present conditions relative to the preparation of teachers of mathematics for grades 7 and 8 of the public elementary schools of the United States. No effort has been made to draw conclusion or to formulate a program for the future.

It is very evident that in proportion to the need there is but little specific preparation for the teaching of seventh and eighth grade mathematics. There is a failure to realize the truth of the statement previously noted that "this (seventh) grade marks the approach of the time when the pupil should pass from mere control and manipulation to understanding and investigation," and to provide the kind of instruction needed to lead the pupil out into the broader field of mathematical knowledge.

The failure of the school authorities to grasp the real meaning of algebra and concrete geometry in the seventh and eighth grades is characteristic of the attitude of the American people toward the public schools. The fear that formal algebra and formal geometry are going to crowd out some more practical subjects is what lies back of such action as that in Chicago. If the necessary general arithmetic and concrete geometry could have been made part of the arithmetic course with no comment upon the relation to the more formal courses of the secondary school, and no introduction of the names by which the later courses are known, possibly the progress toward a more efficient course of study might have been greatly hastened. Only 38 per cent of the cities report any algebra (general arithmetic) in grades 7 and 8 and only 12 per cent report any concrete geometry.

We need in the seventh and eighth grades teachers who have been led to see mathematics as a part of human progress, and who can open the eyes of their pupils to behold the same vision. In the words of President Butler:[1]

Mathematics, therefore, studies an aspect of all knowing, and reveals to us the universe as it presents itself, in one form, to mind. To apprehend this, and to be conversant with the higher developments of mathematical reasoning are to have at hand the means of vitalizing all teaching of elementary mathematics.

[1] Introduction to Smith's The Teaching of Elementary Mathematics, p. xi.

COMMITTEE NO. II. SPECIAL KINDS OF ELEMENTARY SCHOOLS.

GENERAL REPORT.

The investigations of the committee on the teaching of mathematics in special kinds of elementary schools have been confined to a discussion of the work as it is given in trade and industrial schools. There are other special types of schools, notably those that deal with defectives and delinquents that might be considered under this classification. It seemed best to the committee, however, to confine their attention to the group mentioned above. The work done in the schools for defectives is a modification of that commonly done in other schools by means of methods of instruction suitable to the type of mind dealt with. In schools for delinquents, whether day truant schools, parental schools, or reformatories, the work is quite commonly either the ordinary work in mathematics given in elementary or secondary schools, or such as is considered in that part of this report relating to industrial schools.

In the Bulletin No. 11 of the National Society for the Promotion of Industrial Education there is given a descriptive list of trade and industrial schools in the United States. This bulletin was issued in August, 1910. There were at that time more than 140 such schools about which the society was able to gain information. The classification of these schools is as follows:

1. Intermediate industrial, preparatory trade, or vocational schools.
2. Trade schools: Day courses—(*a*) supported by public funds; (*b*) supported by private funds.
3. Technical schools: Day courses—(*a*) supported by public funds; (*b*) supported by private funds.
4. Apprenticeship schools.
5. Evening schools: (*a*) Giving technical courses and supported by public funds; (*b*) giving technical courses and on private foundation; (*c*) courses given by Young Men's Christian Associations; (*d*) giving practical shop courses and supported by public funds; (*e*) giving practical shop courses and on private foundation.
6. Part-time schools.

7. Trade schools for colored races.
8. Correspondence schools.

All of these different classes of schools can be considered as special kinds of elementary schools as contrasted with the ordinary high school. From another point of view they are secondary schools, since in most cases at least six years of elementary school work is required for entrance, and since the courses of instruction offered lead directly to vocational efficiency. The committee has concerned itself with all of these groups of schools in its report even though they may not have been specified in the outline of work proposed, and may not be specifically mentioned in the later discussion.

Some brief statements of the work of these several classes of schools may not be out of place as a preliminary to the consideration of their work in mathematics. In the statements which follow, the characterization of the catalogue of the National Society for the Promotion of Industrial Education mentioned above is followed, and in some cases the exact wording is quoted.

INTERMEDIATE INDUSTRIAL, PREPARATORY TRADE, OR VOCATIONAL SCHOOLS.

These schools take pupils who are 14 years of age and have completed at least six years of the elementary school course. These children are too young to be accepted as apprentices in the trades, and yet have commonly left school to enter upon occupations which promise very little advancement even after several years of service. "The intermediate industrial school has the double aim of turning the attention of its pupils, and of the parents of pupils, to the superior opportunities for independence offered by work in the manual trades, and of giving them such instruction as will enable them after two or three years to enter the trades with advantage. By this means it is designed to retain for a longer period of time the boys and girls who otherwise leave school."

"In schools of this class, the course of study commonly combines bookwork and shopwork in almost equal proportions. The bookwork generally includes English, shop mathematics, industrial history, and civics, together with the elements of physics and chemistry. The shopwork is usuall confined to a limited choice of preparatory trade work for each sex. Beginning with general industrial practice the work is more specialized toward the end of the course and carries the pupil to a point at which he will be able to enter a skilled trade or a factory industry as an apprentice possessing a general knowledge of the quantities involved and a background of scientific knowledge that will open the way for comparatively rapid promotion." Examples of schools of this type are the following: Secondary Industrial School of Columbus, Ga.; New Bedford Industrial School, New Bedford, Mass.; Lawrence Industrial School, Lawrence, Mass.; Newton Inde-

pendent Industrial School, Newton, Mass.; Albany Vocational School, Albany, N. Y.; Hudson Industrial School, Hudson, N. Y.; New York City Vocational School for Boys, One hundred and thirty-eighth Street and Fifth Avenue; Rochester Factory School, Rochester, N. Y.; Rochester Shop School, No. 26, Rochester, N. Y.; Industrial School, Schenectady, N. Y.

TRADE SCHOOLS.

Trade schools which offer day courses are of two distinct types. Some have very short courses like the Baron de Hirsch Trade School, and the New York Trade School, where very little attention is given to instruction in English, mathematics, or science. In these schools the aim is to prepare pupils for actual work in the trades in the shortest possible time. After a course of from four to six months the student can frequently secure employment as a helper, and, because of the work done in school and his knowledge of the theory of the trade, he can commonly advance rapidly to the grade of journeyman. The trade schools giving longer courses frequently give very thorough instructions in English, mathematics, and science. The Williamson Free School of Trades and the Wilmerding School of Industrial Arts devote a considerable proportion of time to such instruction.

The Manhattan Trade School for Girls, recently established in New York City, and similar schools in Boston and in certain other cities, are especially interesting. We have long been accustomed to the idea of trade schools for boys, but have until recently neglected to realize that there is the same necessity for training and instruction for girls. The results already achieved by girls' trade schools prove that, as in the case of boys, the trade school prepares for a type of work in which a living wage is secured and in which there is opportunity for advancement. The list of trade schools given in the bulletin of the National Society for the Promotion of Industrial Education follows:

SUPPORTED BY PUBLIC FUNDS.

The Manhattan Trade School for Girls, New York City; State Trade School, New Britain, Conn.; Worcester Trade School, Worcester, Mass.; Yonkers Trade School, Yonkers, N. Y.; Portland School of Trades, Portland, Oreg.; Philadelphia Trades School, Twelfth and Locust Streets, Philadelphia, Pa.; Milwaukee School of Trades for Boys, Milwaukee, Wis.; Girls' Trade School, 620 Massachusetts Avenue, Boston, Mass.; New York Trade School for Girls, Syracuse, N. Y.; Milwaukee School of Trades for Girls, Eighteenth and Wells Streets, Milwaukee, Wis.

SUPPORTED BY PRIVATE FOUNDATION.

Wilmerding School of Industrial Arts for Boys, San Francisco, Cal.; Winona Technical Institute, Indianapolis, Ind.; David Ranken, Jr., School of Mechanical Trades, St. Louis, Mo.; Paterson Silk Textile Institute, Paterson, N. J.; School of Science and Technology of Pratt Institute, Ryerson Street, Brooklyn, N. Y.;

Baron de Hirsch Trade School, 222 East Sixty-fourth Street, New York, N. Y.; New York Electrical Trade School, 39 West Seventeenth Street, New York, N. Y.; New York Trade School, First Avenue between Sixty-seventh and Sixty-eighth Streets, New York, N. Y.; Girard College, Philadelphia, Pa.; Williamson Free School of Mechanical Trades, Williamson School, Delaware County, Pa.; Miller School, Albemarle County, Va.; Hebrew Technical School for Girls, Second Avenue and Fifteenth Street, New York, N. Y.; Manhattan Trade School for Girls, 209-213 East Twenty-third Street, New York, N. Y.; Ezra F. Bowman Technical School, Lancaster, Pa.; Omaha Watch Repairing, Engraving and Optical Institute, Omaha, Nebr.; Horological Department of Bradley Polytechnic Institute, Peoria, Ill.; College of Horology, Broad and Somerset Streets, Philadelphia, Pa.; St. Louis Watchmaking School, St. Louis, Mo.; Waltham Horological School, Waltham, Mass.; Coyne National Trade School, 1701 N. Ashland Avenue, Chicago, Ill.; Master Plumbers Trade School, St. Louis, Mo.

TECHNICAL SCHOOLS: DAY COURSES.

"The schools included under this classification are of secondary grade and do not give instruction of higher engineering rank. The courses are designed to give the pupil such acquaintance with the scientific and mathematical principles underlying commercial processes as will enable him to qualify in time for the work of foreman, master mechanic, inspector, etc. Special attention is called to the group of textile schools, which are not trade but technical schools, in which both the practical and the theoretical aspects of textile manufacture are presented."

The following schools are supported by public funds: Bradford Durfee Textile School, Fall River, Mass.; Lowell Textile School, Lowell, Mass.; New Bedford Textile School, New Bedford, Mass.; Technical High School, Cleveland, Ohio; Louisiana Industrial Institute, Ruston, La. Supported by private funds: California School of Mechanical Arts, San Francisco, Cal.; Bliss electrical School, Takoma Park, Washington, D. C.; Lewis Institute, Madison and Robey Streets, Chicago, Ill.; Hebrew Technical Institute, 36 Stuyvesant Street, New York, N. Y.; Pratt Institute, Brooklyn, N. Y.; Webb's Academy for Shipbuilders, Fordham Heights, N. Y.; Ohio Mechanics' Institute, Cincinnati, Ohio; Drexel Institute of Art, Science, and Industry, Philadelphia, Pa.; Philadelphia Textile School, Philadelphia, Pa.; Carnegie Technical Schools, Pittsburgh, Pa.; High Schools of Practical Arts for Girls, Boston, Mass.

APPRENTICESHIP SCHOOLS.

Apprenticeship schools are provided by a number of large industrial corporations for the education of boys who are learning their trade. In these schools there is a close correlation of the theoretical instruction and the practical shopwork. Through training in mathematics, drafting, English, and science, there are recruited from the ranks of those who work in the shops a group of

men much more capable than the ordinary worker. Sometimes from this group there are chosen those who occupy the higher supervisory positions. The movement is an indication of the desire of these corporations to supplement the work ordinarily done in the shop during the period of apprenticeship with a type of theoretical work which will develop a more intelligent workman. The fact that in most of these schools shop time is taken for the class work, and that this time is paid for at the regular rate, is an indication of the value attached to such exercises. The following is a partial list of such schools: Apprenticeship System of the New York Central lines; Apprenticeship Schools of the Santa Fe Railroad System; Apprenticeship School of the General Electric Co., West Lynn, Mass.; Yale & Towne Manufacturing Co.'s Apprentice School, Stamford, Conn.; School for Apprentices of the Lakeside Press, Chicago, Ill.; Solvay Process Co.'s Apprentice School, Syracuse, N. Y.; School of Printing of the North End Union, Boston, Mass.; Drawing School of the American Steel & Wire Co., Worcester, Mass.; Fore River Shipbuilding Co.'s Apprentice System, Quincy, Mass.; Ludlow Textile School, Ludlow, Mass.; Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.; John Wanamaker Commercial Institute, New York, N. Y., and Philadelphia, Pa.

EVENING SCHOOLS.

Evening schools are more closely related to the apprenticeship schools than to any other of those already mentioned, since the students are recruited almost wholly from among those who work at the various trades during the day. They are intended to give a type of education, both theoretical and practical, which will give the worker at the trade a better understanding of his work and of its relationship to the industry toward which his work contributes. The tendency of shopwork is to confine the worker to one very small part of the whole industry and hence to give him very little appreciation of the significance of his labor, either in relation to the other processes in the same industry or to the industry in the larger social group. These evening schools, commonly called industrial improvement schools or continuation schools, follow a course of study not unlike the corresponding day schools. The following schools are of this type and give technical courses, and are supported by public funds: The Evening Industrial School, Boston, Mass.; Public Evening Trade School, Lowell, Mass.; Bradford Durfee Textile School, Fall River, Mass.; Lowell Textile School, Lowell, Mass.; New Bedford Textile School, New Bedford, Mass.; Evening Industrial School, Pittsfield, Mass.; Evening School, Little Falls, N. Y.; Stuyvesant Evening Trade School, 345 East Fifteenth Street, New York City; Brooklyn Even-

ing Trade School, Brooklyn, N. Y.; Technical High School, Cleveland, Ohio. Schools giving technical courses and on private foundation are as follows: Lewis Institute, Chicago, Ill.; Franklin Union, Berkeley and Appleton Streets, Boston, Mass.; Lowell Institute School for Industrial Foremen, Boston, Mass.; Technical School, Newark, N. J.; Cooper Union for the Advancement of Science and Art, Astor Place, New York City; Mechanics' Institute, 20 West Forty-fourth Street, New York City; School of Science and Technology of Pratt Institute, Brooklyn, N. Y.; Mechanics' Institute, Rochester, N. Y.; Ohio Mechanics' Institute, Cincinnati, Ohio; Casino Technical Night School, East Pitsburgh, Pa.; Drexel Institute, Philadelphia, Pa.; Franklin Institute Night School, Philadelphia, Pa.; Philadelphia Textile School, Philadelphia, Pa.; Spring Garden Institute, Philadelphia, Pa.; School for Apprentices and Journeymen of the Carnegie Technical Schools, Pittsburgh, Pa.; Virginia Mechanics' Institute, Richmond, Va.

Throughout the United States the Young Men's Christian Associations have entered the field of industrial education. The work of the education department in city, railroad, and industrial Young Men's Christian Associations has become one of the most important of the activities of these organizations during the past few years. According to the report of the National Society for the Promotion of Industrial Education, there are now "2,250 paid teachers who give instruction to over 50,000 employees, men and boys, two nights per week for half the year, in a total of 140 different commercial and vocational subjects. Included in the above lines of class work are mechanical, architectural, and freehand drawing, design, shop mathematics, physics, electricity, chemistry, shopwork in wood and metal, plumbing, etc. This industrial work is not designed to prepare for the higher engineering lines, but on the contrary to furnish opportunities for industrial improvement, either by broadening the pupil's knowledge of a trade or by instructing him in the technical foundations of efficient and intelligent trade work." These associations are scattered throughout the United States. There are more than 60 associations, enrolling 50 or more students in trade and industrial subjects.

Schools giving practical shop courses and supported by public funds are found in the following cities: Cambridge Evening Industrial School, Cambridge, Mass.; Evening School, Chicopee, Mass.; Evening School of Trades, Springfield, Mass.; Long Island City Evening High and Trade School, Long Island City, N. Y.; New York Evening High School for Women, East Forty-second Street, near Third Avenue, New York City.; Stuyvesant Evening Trade School, New York, N. Y.; Philadelphia Trades School, Philadelphia, Pa. The following schools give practical shop courses and on private foundation:

Massachusetts Charitable Mechanic Association Trade School, 111 Huntington Avenue, Boston, Mass.; David Ranken, Jr., School of Mechanical Trades, St. Louis, Mo.; Five Points Italian Trade School, 155 Worth Street, New York City; Hebrew Technical Institute, New York City; New York Electrical Trade School, 39 West Seventeenth Street, New York City; New York Trade School, First Avenue, between Sixty-seventh and Sixty-eighth Streets, New York City; Manhattan Trade School for Girls, 209–213 East Twenty-third Street, New York City; School of Science and Technology of Pratt Institute, Brooklyn, New York City; St. George's Evening Trade School, 505 East Sixteenth Street, New York City; Sixty-third Street Evening Trade School of the Children's Aid Society, New York City; Drexel Institue of Art, Science, and Industry, Philadelphia, Pa.; Hebrew Education Society, Tenth and Carpenter Streets, Philadelphia, Pa.

PART-TIME SCHOOLS.

"The part-time plan, as carried on at Fitchburg, Mass., is an arrangement by which boys in the high school give half of their time to work for wages in the commercial shops in the city and half to school work. Such a plan is designed to furnish the pupil with the theoretical training necessary to his advancement in industrial life, while at the same time providing practical experience and wage return. The school work is under the direction of the board of education of Fitchburg and is supported by public funds. The classroom instruction articulates with the grammar school, and implies regular high-school requirements. For this part of the work the regular school building is used. For the first year the boy devotes his entire time to school work, and for the next three years equal groups of boys alternate weekly between shop and school, so that one group is always at work in the shops and one in the school. On Saturday morning of each week the boy who has been at school for that week goes to the shop in order to become familiar with the work upon which his alternate is working so as to be ready to take it up on Monday morning when the shop boy returns to school. Boys are paid for the work done in the shops at the rate of 10 cents an hour for the first year, 11 cents for the second year, and 12½ cents for the third year, or a total of $552.75 for the three years." Other part-time schools are found at Beverly, Mass. (The Beverly Industrial School) and at Cincinnati, Ohio (The Cincinnati Continuation School). All of these schools are publicly supported. Doubtless other schools of this type will be open in considerable numbers in the very near future.

TRADE SCHOOLS FOR THE COLORED RACES.

Some of the best industrial work that is done in the United States is to be found in schools for the education of Indians and negroes.

In all of these schools the industrial work is accompanied by literary instruction. In some of the schools as much as half of the time is devoted to industrial training. In some of the schools the relationship between industrial and the ordinary academic work is exceptionally well handled. This is peculiarly true in the subject of mathematics. The problems of the shop and farm are quite commonly treated in the classes in arithmetic, algebra, and geometry. "Normal instruction, instruction in the domestic arts and sciences, and giving each pupil such an understanding of a manual trade or agriculture as will make him financially independent and hence self-respecting—all these features together represent the aim of such schools as Hampton and Tuskegee Institutes and the other institutions in the list." Schools of this class are as follows: Hampton Normal and Agricultural Institute, Hampton, Va.; Tuskegee Normal and Industrial Institute, Tuskegee, Ala.; United States Indian School, Carlisle, Pa.; Christiansburg Industrial Institute, Cambria, Va.; Voorhees Industrial School, Denmark, S. C.; Schofield Normal and Industrial School, Aiken, S. C.; Manassas Industrial School, Manassas, Va.; Mayesville Industrial Institute, Mayesville, S. C.; Okolona Industrial College, Okolona, Miss.; Fessenden Academy and Industrial School, Fessenden, Fla.; Montgomery Industrial School for Girls, Montgomery, Ala.; Colored Industrial and Normal School, Salisbury, Md.; Manual Training and Industrial School for Colored Youth, Bordentown, N. J.

CORRESPONDENCE SCHOOLS.

Many persons who are regularly employed in the industries have sought help through the technical work which is offered by correspondence schools. With the persevering student, they have been able to conduct courses in a very great variety of technical subjects, and have in this way furnished the theoretical background which has made possible a wider outlook and a greater degree of efficiency for those who are regularly employed. The following schools are representative of the best type of this class: American School of Correspondence, Fifty-eighth Street and Drexel Avenue, Chicago, Ill.; International Typographical Union Course in Printing, 130 Sherman Street, Chicago, Ill.; International Correspondence Schools, Scranton, Pa.

The work of these schools is considered in another report to the International Commission on the Teaching of Mathematics.

SUBCOMMITTEE 1. INDUSTRIAL CLASSES IN PUBLIC SCHOOLS.

The subcommittee reporting on industrial classes in public schools included in its inquiry all schools supported and controlled by the municipality through its board of education. There were included, therefore, graded schools having a course of study of from seven to

nine years in length, the technical and manual-training high schools, usually not coeducational, and the ordinary coeducational academic high schools. Evening schools of either elementary or secondary grade, as well as trade schools supported by the municipality as a part of the public-school system, were also considered. The term "industrial classes" was interpreted broadly to mean classes in manual training, domestic art, domestic science in either of the grades or high school, as well as classes in high-school commercial courses. Classes in vocational work in any of the schools named above were also included.

METHOD OF COLLECTING DATA.

A questionnaire was sent to 150 high-school principals and city superintendents in the larger cities of the United States, asking for data on the subject of "Industrial classes in public schools," and the relation of the work of these classes to the work in mathematics. While the response to this questionnaire was not as general or as full as could be desired, yet many superintendents cheerfully responded. It is largely on the basis of these replies that this report has been prepared.

KINDS OF INDUSTRIAL WORK GIVEN IN PUBLIC SCHOOLS.

Industrial work is given in public schools: First, in classes in manual training, domestic art, and domestic science throughout the grades of city school systems; second, in classes in manual training, domestic science, and commercial branches, conducted in the ordinary free academic high schools; third, in technical, manual-training, and commercial high schools; fourth, in evening schools of elementary or secondary grade; fifth, in vacation schools; sixth, in trade schools supported by the municipality and under the control of the board of education. In a few cases vocational classes are maintained in the free academic high schools; and in a very few instances in this country industrial work is given by a cooperation of public schools, and shops and factories. In these cases the boys work part time in the shop or factories and part time in the school.

CULTURAL INDUSTRIAL WORK.

In all parts of the United States a constantly increasing number of public schools have provided instruction in manual training, domestic art, domestic science, and commercial branches. The manual training consisting of whittling and bench work in wood is usually the first form of industrial work to be introduced. The next is sewing and dressmaking, followed by cooking. As public sentiment becomes educated to a more liberal support of this work, it is extended

to all grades below the high school, and also to each year in the high school.

In answer to the question, "What kinds of industrial work are given in your school?" the following reply from one of the large cities in the Middle West is typical: "Strictly speaking, we do not give any industrial work in our public schools unless you would call manual training, domestic art, and domestic science, industrial work. (*a*) Handwork is taught the children in the first four grades (primary). This handwork consists of two kinds—first, work to illustrate subjects about which they read; second, independent unrelated exercises. In the fifth and sixth grades, boys are taught knife work by a special teacher, and girls sewing. In the fifth grade we give to this work one and one-half hours each week. In the sixth grade two and one-half hours a week. In the seventh and eighth grades boys are taught joinery and cabinetmaking in specially equipped shops; girls are taught cooking. Half a day a week is given to those subjects in the seventh and eighth grades. (*b*) We are now constructing two manual training high schools which will be ready for occupancy next fall. The manual training in these high schools will not take the form of industrial training, though it may develop into that later. At present in the high schools we have a very complete commercial department which fits pupils for business life and for clerical work. We also teach mechanical drawing, free-hand drawing, and applied design, with some work in the crafts."

From another large city where the high school industrial work was much more extensive, the following reply was received: "In our high schools, sewing, millinery, carpentry, blacksmithing, machine-shop practice, wood turning, architectural drawing, and mechanical drawing are given."

For the most part the lines of industrial education named above furnish the basis for work in the trades rather than direct instruction in the trades.

VOCATIONAL INDUSTRIAL WORK.

Besides the above lines of work there are in some schools strictly vocational classes. Many school boards maintain evening schools of elementary and secondary grade. Much of the work in these schools is vocational. It includes one or more of the following: Millinery, dressmaking, flowermaking, chauffeur's work, wood turning, cabinetmaking, architectural drawing, mechanical drawing, stenography, bookkeeping, and other trades of local interest. Of course the high school commercial courses prepare directly for business life and for clerical positions. Trade schools for girls, evening grammar schools, evening high schools, technical high schools, manual training high

schools are forms of schools in which industrial work is prominent. In all of these two kinds of industrial work are carried on—first, that which furnishes a basis for trade or technical instruction, but which is itself not directly trade instruction; second, strictly vocational work.

COOPERATIVE INDUSTRIAL WORK.

A very interesting experiment is being tried in a few cities in this country—the so-called cooperative industrial work. Under this plan boys work one week in the shops or factories and the next week in the public schools. The work in school is such as will directly affect their efficiency in the shop. This plan gives boys their industrial work in the shops and their academic work in school. The academic work is correlated with the shopwork. A number of other cities are planning to try this experiment.

PUBLIC SCHOOL INDUSTRIAL WORK AND LOCAL INDUSTRIES.

Of the cities from which answers to the questionnaire were received very few stated that the work offered in manual training and domestic science is, in any special way, modified to fit the local industries, while several stated that no such relation exists. The following reply is from one of the large cities of the East: "No special attempt has been made * * * in public school work to fit pupils especially for local industries. We have such a variety of local industries that it would be impossible to train especially for certain ones and not for others."

Where evening and trade schools are maintained, a very definite relationship exists between the trades taught and the local industries. Those trades that are carried on in the city where these schools are located are taught in the schools.

ARE PRESENT CONDITIONS SATISFACTORY.

In general, superintendents seem to feel that the industrial work now given is satisfactory as far as it goes. Many express the opinion that more time should be given to the work and greater diversity should be introduced.

The following reply is from a superintendent who has this point of view. He says: "We do not feel that we have really made a beginning in meeting the demand for industrial education. The crowded condition of our schools prevents any advanced steps in this direction. We aim as soon as possible, however, to establish schools which will give greater training along a number of lines of activities."

Another superintendent writes as follows: "Our present industrial work is not satisfactory, if it is thought advisable to give vocational

training. I am not satisfied yet that vocational training should be introduced into public schools. However, I am open to conviction. We are hoping to try a plan of part-day school and part-day work in the shops and in other vocations. If this plan works out satisfactorily it will mean that we will use the shops and factories of the city for the practical work, while the academic and theoretical work will be given in the schools."

Still another superintendent writes: "We want more work of an industrial nature in the schools we have. We want a thoroughly equipped industrial plant in our high school. We want a school that will give students a thorough training in all departments of the silk industry, our leading industry, and its associated occupations, the manufacturing of silk machinery, etc. We should do something for the young men who follow the iron and steel industries, locomotive building, structural-steel work, etc."

THE MATHEMATICAL CURRICULUM.

It is customary to give a course in arithmetic through the grades, beginning in the first or second grade and extending through the eighth. In the eighth grade a course in constructive geometry is given and sometimes one in elementary algebra.

In the academic high schools having four-year courses the customary course in mathematics is given as follows: One year of algebra is required, and this is usually given in the first year of the high-school course; one year of geometry is also required in most high schools. Sometimes it is a year of plane and solid geometry, given, as a rule, in the third year of the high-school course. Sometimes the one-year course is devoted entirely to plane geometry; then it is usually given in the second year of the high-school course. Arithmetic is generally required in the commercial courses, and in some high schools it is required in other courses also. In addition, in the third or fourth year of the high-school courses a half year of advanced algebra and a half year of solid geometry are offered as electives, and then plane geometry only is required.

The following course in mathematics, given in one of the largest high schools in the Middle West, is typical of the work given in schools of this class: In the first year, algebra; in the second year, plane geometry; in the third year, solid geometry and advanced algebra; in the fourth year, trigonometry. The instructor in mathematics in this school writes as follows: "A modification of this course that would possibly be an improvement is the introduction of elementary solid geometry propositions, and trigonometry notions in a first course in geometry, to be given in the second year; the remainder of solid geometry together with the most difficult plane

geometry propositions, to constitute an advanced course, coming in the third year. This would permit of the introduction of a wide range of extremely interesting problem material into the geometry course."

RELATION OF MATHEMATICS TO INDUSTRIAL WORK.

The general tenor of the replies received indicates that a greater effort is being made to correlate mathematics taught in the grades and in the high school with manual training, domestic science, and other industrial work offered in these schools. As a rule in academic high schools, separate classes in industrial mathematics are not conducted. In evening and trade schools, the mathematics offered is taught frequently by the instructor in the trade. In this case, the mathematics taught is such as will be useful in the given trade. Shop formulas are taught; these formulas are applied to practical problems and the meaning of the formulas is made clear. Emphasis is placed, primarily but not exclusively, on the empirical use of the formulas instead of on the demonstrative side as in high-school mathematics. In the Washington Irving High School in New York City, a technical high school for girls, the course in mathematics follows a course made by the domestic art and commercial departments of the school.

The following replies to the questionnaire will indicate in greater detail what is being done along the line of industrial mathematics and the effect industrial work is destined to have on mathematical teaching.

From a large city in the East is the following reply: "Industrial mathematics is taken up in the Technical Evening High School, and in the vocational schools. In each case, separate classes are held for mathematics, but it is correlated as much as possible with shopwork. The demonstrative side is not emphasized particularly, neither are formulas given empirically. Our instructors try to show the reason why and to make the rules and formulas logical deductions. I have no doubt that with the growth of industrial work in the schools a system of shop mathematics will be evolved, beginning with arithmetic and extending certainly through geometry, which will apply particularly to the industries."

Another superintendent writes: "No work in industrial mathematics is given. Use is made of practical problems and applications of mathematics to the arts and sciences because experience has shown that such problems add interest to the work."

A third superintendent in one of the large Western cities says: "There is nothing special in our course of study known as industrial mathematics. I think, however, that there is need for work of this kind. In connection with manual training, mathematics especially

suited to this kind of work is, of course, used. The same is true in regard to various lines of manual activities offered in the Manual Training High School. There is no doubt that the public school industrial work will tend to make arithmetic, taught in the elementary schools, much simpler and more practical in its character."

A principal of one of the large high schools in the Middle West says: "Shop arithmetic is given with class lectures and work in the shop. Application is emphasized. No attention whatever is paid to demonstration. There is no correlation at present between mathematics and industrial work. In the near future, the mathematical subjects will have to treat their problems and theorems as some part of the world in which we live, and not as something that pupils think exists in imagination. Industrial work will make demands on the pupils which will require them to get their mathematics in organized shape and hold it ready for every emergency."

In a large city in the Middle West the practical side of mathematics in the grades is emphasized by supplying pupils with mimeographed sets of questions and problems such as would arise in buying familiar articles at the meat market, at the grocery department, or at a department store in its various departments. These problems are arranged to bring out the idea of economical buying of articles needed for a meal and of buying articles needed in the various phases of domestic work. Problems are given on the cost of heating and lighting a house by gas and by electricity. In this case, statement is made of the number of feet of gas used, and the number of kilowatts of electricity used, and pupils are required to compute the monthly bill, for these items. Problems are also given relating to the practical work of different industries, such as railroad and foundry problems, pay rolls, cost of house furnishing, etc. Data are given of the amount of money spent by the city in each of the various kinds of city improvements, and the amount of improvement made, for example, the number of miles of street paved in brick, in asphalt, etc. From these data a large number of problems are developed. It is evident that problems of this kind connect directly with the child's home life, with the industrial life, and with the civic life with which he is familiar.

CHANGES NEEDED IN THE MATHEMATICAL CURRICULUM.

The changes needed in the mathematical content and curriculum are along the lines indicated in the foregoing. The principal of one of the largest high schools in New York City says: "The mathematical courses need reformation. What can any sort of mathematics do for the ordinary man, the ordinary woman, the industrial worker, the housekeeper? Get this mathematical desideratum expressed in terms of power and information, and then plan your course in mathe-

matics to produce the outlined purposes. At present the procedure is, 'Every high school has algebra and geometry. We'll have them too.' Neither of these subjects springs out of the needs of the students, nor is it concretely directed toward equipping the students with any useful knowledge or power."

The following quotation sums up the opinion expressed in the large majority of the replies received: "Some of the less essential should be eliminated * * * and more drill given to concrete problems involving home and industrial life."

REQUIRED AND ELECTIVE MATHEMATICS.

The opinion of a large majority of principals and superintendents, answering the questionnaire, is expressed by the following quotation: "Elementary algebra and plane geometry should be required of all. All should have at least this amount of mathematics so as to appreciate the disciplinary and practical value of this branch of human knowledge. Algebra and geometry are essential to a study of the sciences. Higher algebra and solid geometry may with profit be made elective."

The following quotation expresses a more radical view: "If algebra and geometry must come at the beginning of the course, I would make both elective on account of the immaturity of most beginning students. The great modification that I would suggest in high school mathematics would be a shifting of the whole matter at least one year higher in the course, using the first year for language, history, and science subjects."

PROPOSED FUSION OF DIFFERENT BRANCHES OF MATHEMATICS.

The general opinion of those answering the questionnaire is that algebra and geometry should not be fused into one course in mathematics, nor should plane and solid geometry be fused. The following is a typical quotation: "Algebra and geometry should not be fused. By such a plan students will gain very little knowledge of either." Algebra should precede geometry. The step from arithmetic to algebra is much easier than from arithmetic to geometry. Arithmetic and algebra are more closely related. Plane and solid geometry should not be fused.

The opinion is very generally expressed that, while it is undesirable to make the fusions spoken of, a very much closer relationship should exist between the different branches of mathematics offered in the high school.

ORDER OF MATHEMATICAL STUDIES.

The quotations given above and many others of similar nature that could be given, express the prevailing opinion that algebra should

precede demonstrative geometry, though a course in constructive geometry should precede algebra. Plane geometry should precede solid geometry.

THE UTILITARIAN SIDE OF MATHEMATICS IN THE PUBLIC SCHOOLS.

"In what ways is stress placed on the utilitarian side of mathematics?" was one of the questions asked. This is a question to which few satisfactory replies were received. The principal of one of the largest technical high schools in the Middle West says: "Use is made of practical problems and applications of mathematics to the arts and sciences because experience has shown that such problems add interest to the class work."

Another principal says: "In the grades no special stress is placed on the utilitarian side except that we attempt to find worthy concrete problems. In the Manual Training High School shopwork is made the basis of many problems in arithmetic and geometry. The commercial course in the high school aims to be altogether practical."

A superintendent of a large eastern city says: "Commercial arithmetic is studied as a regular subject in the business course, and all work in manual training is from drawings and plans, requiring mathematical computations."

Another superintendent states: "Material for problems in all grades is drawn from home and store and general outside experience. Printed forms, bills, and blanks of various kinds of business are used in the classroom, tax bills, gas bills, etc. Pupils are required to compute the customary discount on these bills."

MATHEMATICS AND APPLIED SCIENCES.

The replies indicate a general tendency to use in the work in mathematics problems from the applied sciences, models, etc., to make the work more effective. The following are quotations from the answers to this phase of the inquiry: "Problems of parallelograms of forces, falling bodies, immersed solids, solids made in the manual training department are used." "Problems in elementary mechanics, cardboard models, and similar devices are used." "Sectional models should be used as foundation work in mensuration in the grades and in the high school. All instruments and machines from which problems are drawn should be, so far as possible, studied, and from these the results should be verified."

CORRELATION OF DIFFERENT LINES OF MATHEMATICS.

Relatively few schools indicate any considerable attempt to correlate the different lines of mathematical work in school. A very large number of schools make no reply to this phase of the inquiry. Some report that no correlation is attempted. A few report that

work along this line is carried on: "Algebraic methods are used in geometry work; algebraic results and formulas are illustrated geometrically; logarithmic computations of volumes is made in solid geometry." "Algebra and geometry, while not taught together, are made interdependent as much as possible."

DRAWING AND GEOMETRY.

Is the fundamental relation between drawing and geometry emphasized? If so, how? To this question few answers were received. The following are some of them: "Yes. In mechanical and architectural drawing." "The course in mechanical drawing, given in the first year, is introduced by the graphical solution of a number of geometric construction problems." "No. The drawing teacher teaches perspective, for instance, which is a direct application of geometry and optics, without any reference to either subject. I doubt if any drawing teacher ever was taught the connection." "The fundamental relation between geometry and drawing is emphasized both in the grades and in the high school, in manual training, in arithmetic, in geometry, and in mechanical drawing." "Very strongly. We give an eight weeks' course in preliminary geometry, and insist on accurate drawings all the time." "Mechanical drawing is applied geometry, and its relations are made use of perforce in all mechanical drawing and drafting for industrial work." These quotations serve to illustrate that in spite of some divergence in opinion the importance of this phase of the work is coming to be recognized in schoolroom practice.

EDUCATIONAL MUSEUMS.

Very few of the cities heard from report that they are equipped with a mathematical museum. Very many of those who replied indicate a belief in the value of such a museum, as the following quotations show: "Undoubtedly a collection of models, instruments, pictures, etc., would be of great value." "We believe it has some value, but it is not essential." "It is too bad that we have not such a museum." "Illustrative material is always of value. It stimulates interest and self-activity." Occasionally a pessimistic note is heard as the following: "I never saw an educational museum that was worth anything, chiefly because it was so administered that the teachers would not take the trouble to use it."

CONCLUSION.

These replies, which were received from cities in all parts of the United States, indicate a strong tendency to increase the amount and variety of industrial work offered in public schools.

Where trades are taught instruction in shop mathematics is given usually by the instructor in the trades. The work as given in these classes in shop mathematics is adapted directly to the trades taught. In the regular mathematical work in the public schools no great change in the mathematical curriculum is demanded.

The effect of the industrial work is, therefore, to introduce into all lines of public-school mathematics a larger number of problems related to the industrial work done in school, to the industries carried on in the community, to the actual life of the child and his home, and to the civic enterprise of the city.

SUBCOMMITTEE 2. CORPORATION INDUSTRIAL SCHOOLS.

In corporation industrial schools the type of work done in mathematics varies with the demand for mathematical knowledge in the work carried on by the corporation. In one school, as, for example, the John Wanamaker Commercial Institute, the work will consist mainly of commercial arithmetic. In another school the mathematical curriculum will extend to a knowledge of algebra, geometry, and possibly trigonometry because of the need for understanding formulæ which must be applied in machine shops. In these schools, as has been suggested above in the discussion of apprenticeship schools, there are found boys who have had from six to eight years of elementary school work, with an occasional boy who may have had some work in high school. With this rather meager equipment, and with the large need for mathematical knowledge which must be applied to shopwork, it becomes apparent that the problem of the shop school is largely one of getting before the class problems which deal with actual shop conditions and teachers who understand shopwork, as well as theoretical mathematics.

Our standard textbooks deal with mathematics in a general way, covering many subjects which have no connection with manufacturing industries. This being the case we can not use the existing textbooks for problem work, and therefore it falls upon the instructor to write the problems required. These problems must, of course, follow in order the usual textbook classification. Since each kind of industry requires problems adapted to its production, no one book can provide problems for all industries requiring trade knowledge. However, one textbook can cover sufficiently the industries representing the manufacture of machinery, engines, and electrical equipment.

The problems must not only follow textbook classification, but they must be in fact what they purport to be, namely, shop problems. In the writing of these problems it is absolutely necessary that the

instructor should be a man who has had shop experience and one who is familiar with the various departments and in touch with factory conditions.

These problems will involve not only a knowledge of the machines in use but an intimate knowledge of the factory production, stock departments, and piecework rates. This knowledge is necessary because the shop boy is quick to note an error in dealing with factory work, and he is equally quick to lose confidence in the instructor who shows a lack of shop experience.

The work in arithmetic, algebra, geometry, and trigonometry in schools of this type is not sharply differentiated as in the ordinary school. In order to indicate something of the nature of the relationship which exists among these subjects, as well as to give some idea of the subject matter covered and the method of instruction, the program of work carried out under the direction of Maj. A. W. Lowe, superintendent of educational classes for the General Electric Co., West Lynn, Mass., is given in the following pages:

ARITHMETIC.

Having written the problems, the next step is to have the apprentices work out the values. One method of teaching the above subject is to have the problem given in the usual way; then substitute literal values for the numerical values previously given and put the problem in the form of an algebraic statement; then substitute a set of numerical values for the literal quantities and solve the problem.

I give herewith specimen examples:

Problem 1. Brass castings are worth 23.25 cents per pound; cast-iron castings, 3.75 cents per pound; malleable-iron castings 4.68 cents per pound. During one week 5,980 pounds of malleable-iron castings, 8,760 pounds of brass castings, and 21,900 pounds of cast-iron castings were shipped. What was the value of the castings sent out?

Problem 2. Brass castings are worth (a) cents per pound; cast-iron castings, (b) cents per pound; malleable-iron castings, (c) cents per pound. During one week (d) pounds of malleable-iron castings, (e) pounds of brass castings, and (f) pounds of cast-iron castings were shipped. What was the value of the castings sent out?

Problem 3. Using the literal quantities of problem 2, let (a) = 27.5, (b) = 3.8, (c) = 4.38, (d) = 6,830, (e) = 7,240, and (f) = 2,250. What was the value of the castings sent out?

Problem 4. With a good quantity of coal the average coal consumption is $1\frac{3}{4}$ pounds per indicated horsepower per hour for a 1,400 H. P. triple-expansion engine running condensing. How long will it take to consume 250 tons of coal, providing the engine averages a $14\frac{7}{8}$-hour run for each 24 hours?

Problem 5. With a good quantity of coal the consumption is (a) pounds per indicated horsepower per hour for a (b) H. P. triple-expansion engine running condensing. How long will it take to consume (c) tons of coal, provided the engine averages a $16\frac{1}{2}$-hour run for each 24 hours?

Problem 6. In problem 5 let (a) = $1\frac{3}{4}$ pounds, (b) = 1,250 H. P., and (c) = 135 tons. How long will the supply of fuel last?

It is an excellent method of introducing to the apprentice the first steps in algebra, and my experience shows that the boy by this method grasps the idea of algebra much more quickly than by the usual "cut and dried" methods.

ALGEBRA.

A portion of the arithmetic class period should be set apart for the study of elementary algebra.

It may be argued by some that the study of algebra is not necessary in an industrial education. To such I will say that the study of algebra is necessary to a proper understanding of the formulæ which the boy will be called upon to use in his later work.

In the teaching of this subject great care must be exercised by the instructor in that his explanation of elementary principles must be so clear and concise as to enable the boy to understand the very foundation of algebra. Much attention is given to the use of the formula and the simple equation.

PLANE GEOMETRY.

In the teaching of plane geometry we give the boy instruction relating to the line, and plane figures. He is held to a rigorous explanation of the theorem and the class is not allowed to pass to the next theorem until the one under discussion is thoroughly understood. Wherever possible, the attention of the boy is called to a practical application of the principle involved. This arouses the interest of the boy, for the subject then has a meaning and its application is tangible and not imaginary.

In connection with the geometry the subject of mensuration is taught. In problems involving weight the boy first calculates the weight and then checks this calculation by weighing the article.

If there be a difference in the two results then the boy must prove whether or not the calculation is correct. He finally decides that the error is due to his mistakes in the calculation. With the area of a plane surface a similar method is followed and the boy must measure the surface and compare the result with his calculated area.

A few specimen problems are given below:

Problem 1. A cylindrical oil tank is to be made 30 inches in diameter and long enough to hold 200 gallons. Find the length.

Problem 2. The handwheel of a rheostat is 5½ inches inside diameter and 7½ inches outside diameter and cross section 1 inch in diameter. The hub is 1½ inches in diameter by 1½ inches long and the bore is one-half inch in diameter; connecting the hub and hand part are 4 spokes each one-half inch in diameter. Find the weight, if made of C. I.

Problem 3. The concentric diffuser of an arc lamp is conical shape, 40 inches in diameter and 8 inches high. If the upper rays of light strike the surface 6 inches from the base, find the amount of reflecting surface used.

Problem 4. One part of a friction brake is of the shape of the frustum of a cone. The outer surface of the frustum is 8 inches in diameter and the top is 6 inches in diameter; the height is 1½ inches. What is the angle at the center for the development of the leather, and what is the radius for the base and for the top?

TRIGONOMETRY.

This ends the subject of mathematics as pure mathematics. It has been proven repeatedly in shopwork that a working knowledge of trigonometry is one of the most valuable assets a shopman can possess. On large, and some-

times small, layout work a knowledge of this subject is absolutely necessary, for oftentimes an exact dimension must be given and the work must be finished to that dimension.

Our boys are given instruction in trigonometry through the solution of the oblique triangle. Not all the examples or problems are given to the class, but enough are given to enable the boy to handle any problem which he will be called upon to solve in the execution of his shopwork.

He is given thorough instruction in the fundamental principles of the subject, and his knowledge of these principles is thoroughly tested.

A few specimen problems are given below:

Problem 1. Three holes are so located on a casting that the lines connecting their centers form a right triangle. A is an acute angle of 28 degrees and the hypothenuse (c) is 11 inches. Find the sides a and b.

Problem 2. A cast-steel exhaust flange is 20 inches in diameter, is $1\frac{1}{4}$ inches thick, and has 8 bolt holes nine-sixteenths inch in diameter and exhaust opening 6 inches in diameter. What is the surface area? What is the weight? What is the distance between the centers of any two consecutive bolt holes? If one-eighth inch was allowed for machining, what is the original weight of the casting?

Throughout such a course as is outlined above, the real test of the boy's knowledge of his theoretical work is found in his ability to apply the principles supposedly mastered, to what are usually termed practical problems. In the work at West Lynn the boy's knowledge of mathematics is applied to mechanics, power of transmission, strength of materials, calculating machine parts, magnetism, and electricity.

Mr. Gardner, of the School for Apprentices of the New York Central Lines, characterizes their work in the following language:

Stress is laid upon the practical and commercial side of the mathematical instruction. Every example is clothed in the language of the shops and is illustrated by actual practice in the boy's daily work. For example, the boy learns ratio and proportion by figuring the change gears for cutting different screws in his lathe, and the principles of leverage are demonstrated by the throttle and reverse lever on the locomotive and the brake rigging on the car. No attempt is made to teach subjects promoting general culture. The system is not philanthropic, but is a hard and fast business proposition firmly fixed upon a paying basis.

Things are not taught that are interesting to the teacher, but those that are useful to the student. The apprentice needs a working knowledge of elementary mathematics, but the vital point is that it should be elementary and that it must be a working knowledge. The problems are not regarded as affording a complete program of study in itself, but always form part of a regular course planned in accordance with the trade the boy is following.

Arithmetic, algebra, geometry, physics, and practical mechanics are fused so completely that the student knows no study by any name but arithmetic. It has not been found expedient for skilled mechanics in railroad service to acquire a knowledge of higher mathematics.

All conventional limits between all branches of mathematics are suppressed. Mathematics is taught by lesson sheets. Each boy takes two sheets of problems home per week, each sheet containing an average of seven problems. In the

classroom and laboratory two sheets of practical problems capable of actual demonstration are finished per week. Bright boys may do as many more at home or in the classroom as they wish.

The boy is taught by the instructor and his assistant, and is not told how to do examples, but is asked how to do them. Groups of boys are instructed in the laboratory by the shop or classroom instructor. Instruction is very informal, and friendly discussions are encouraged between the boys and the teacher, consistent with order and discipline.

Leverage models for problems in all classes of levers, racks containing gears, and lathes for studying the properties of gearing, wheel and axle model, pulley blocks, inclined plane, screw jack, etc., are used. A small upright engine is used in the laboratory for teaching valve setting. Models of the Stephenson and Walschaert valve gears are studied and small scale models of engine wheels and frames are employed for teaching boys the practical way of laying out keyways for axles and eccentrics and for setting the shoes and wedges which are used in the main driving-wheel journals of locomotives. A small tension and compression machine is used for a course in strength of materials.

Sample sheets of home problems used in the apprenticeship schools of the New York Central lines follow:

If a raise of pay of 10 per cent is made in a man's wages it means that one-tenth of the present amount will be added to it. If his rate is \$2.50 a day one-tenth of this is 25 cents, and his new rate is \$2.75.

Suppose now our man getting \$2.75 a day was cut 10 per cent: what would he receive? 2.75+10/100=0.275, and \$2.75−0.275=\$2.47½, which is less than he got in the first place before he was raised 10 per cent. We thus see that a 10 per cent raise is less in amount than a 10 per cent cut. This is because our base for the cut is a larger sum than the base for the raise.

When we say an engine or motor is 85 per cent efficient, we mean that we are getting 85 parts of work out of a possible 100. When we say a certain quantity of coal contains 65 per cent carbon and 1 per cent sulphur, we mean that 65/100 of any amount of this coal is carbon and 1/100 is sulphur. We do not mean that 65 pounds are carbon and 1 pound is sulphur.

RATIO AND PROPORTION.

1. If a planer has a cutting speed of 30 feet per minute and a return speed of 147 feet per minute, what is the ratio of the cutting speed to the return speed?

2. What ratio is the area of the steam port to that of the exhaust port on an engine having a steam port of 24 square inches and an exhaust port of 36 square inches?

3. The dead load of a 50-ton freight car, consisting of the weight of the car empty and the trucks, is 42,600 pounds. The live load, or the weight of the freight carried, is 108,000 pounds. What is the ratio of the live to the dead load?

4. If the car in the above problem had only carried 87,400 pounds of freight, what would have been the ratio of the live to the dead load?

5. What is the ratio of the dead to the live load in a passenger coach where the car and trucks weigh 90,450 pounds, and the passengers weigh 4,500 pounds?

6. On a 36-inch planer the ratio of the cutting speed to the return speed of the table is 1 to 2.94. With a return speed of 50 feet per minute, what is the cutting speed?

BOILERMAKER'S HOME PROBLEMS.

1. How much will it take to shear a three-fourth inch steel rivet if the shearing strength of the steel is rated at 45,000 pounds per square inch?

2. The tensile strength of boiler plate is 55,000 pounds per square inch. How many pounds would it take to pull apart a piece of plate 24 inches wide by three-fourths inch thick?

3. Two one-half inch plates A and B (fig. 27) are joined by two 1-inch rivets in a single-riveted lap joint. Suppose the plates were pulled part, the plate B tearing between the rivets on the line X X X. If the tensile strength of the

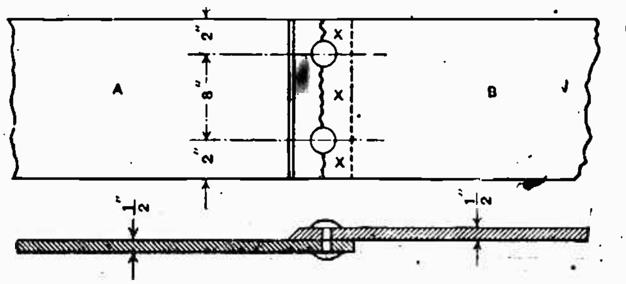


FIG. 27.

plate is 55,000 pounds per square inch, what was the pull in pounds which tore the plate?

4. With the same plates as in the last problem, what pull would tear the plate if the rivets were three-fourth inch instead of 1 inch?

5. If the shearing strength of the rivet iron (fig. 27) is 45,000 pounds per square inch, how much would the two 1-inch rivets stand before they would be sheared off? How much would it take to shear them if they were only three-fourth inch in diameter?

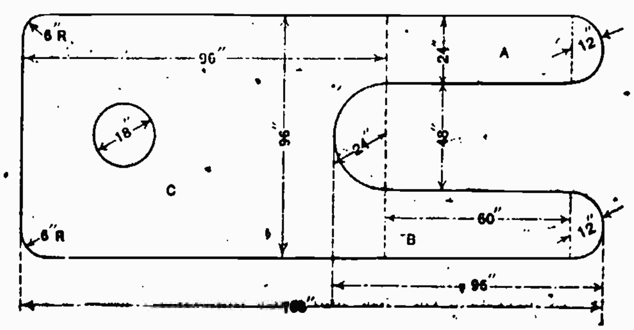


FIG. 28.

6. In the joint shown in fig. 27, if the rivets are 1 inch diameter and the plate one-half inch thick, which will give way first, the rivets or the plate?

GENERAL CLASSROOM PROBLEMS.

Fig. 28 shows the top sheet of a tank made of one-fourth inch steel plate. Divide the figure into 3 rectangles, as shown by the dotted lines. Use Machine Shop Arithmetic, page 137, for all circular areas.

Find area of rectangle A in square inches; find area of rectangle B in square inches; find area of rectangle C in square inches; find area of manhole in square inches; find area of 24-inch half circle in square inches; find area of two 12-inch half circles in square inches; find area of two 6-inch corners in square inches; subtract the two corners, the 24-inch semicircle and the manhole from rectangle C, then add this area to A and B and then add to these the 24-inch circle made by the ends of the two legs; what is area of finished plate in square inches; what is area of finished plate in square feet; what is weight of finished plate in pounds?

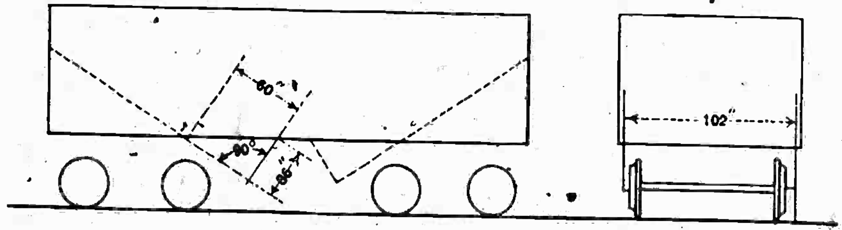


Fig. 29.

Find the weight of coal which is contained in the two hoppers of the hopper-bottom gondola car shown in Fig. 29. Find the area of the triangular end of one hopper and multiply this by the length of the hopper. The shape of the hopper is a triangular prism.

Remember that 1,728 cubic inches make 1 cubic foot and that 1 cubic foot of coal weighs 45 pounds.

Consider the top side of the hopper to be flush with the floor of the car.

Area of two triangular ends, square inches; volume of two hoppers, cubic inches; volume of two hoppers, cubic feet; weight of coal in two hoppers, pounds.

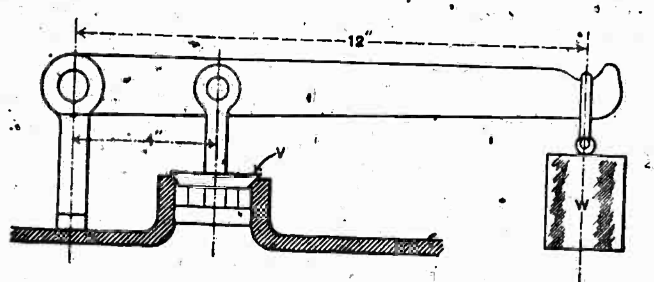


Fig. 30.

In all leverage problems the first and most important thing is to find and locate the fulcrum. The fulcrum is the point which determines the moment arms from which we get the required answer.

The moment arm is always the distance from the fulcrum to the force or weight.

1. A safety valve V on a stationary boiler is loaded with a 50-pound weight at W. (Fig. 30.) Find the total steam pressure on V necessary to open the valve.

2. If the weight was changed to 75 pounds and the distance from the center line of the valve to the fulcrum was made 3 inches, how much steam pressure would it take to open the valve?

3. A stationary boiler is to carry 50 pounds per square inch steam pressure. The safety valve is 2 inches in diameter. If the lever is designed as in Fig. 30, how heavy a weight must be used? Give answer to nearest even pound.

4. What class of lever is shown by Fig. 30.

1. Fig. 31 shows a tender-brake rigging operated by an 8 by 12 inch air cylinder. If the piston moves 8 inches, how far will the push rod move?

2. Draw the similar triangle diagram for problem 1 on the blackboard and mark plainly all dimensions and distances moved.

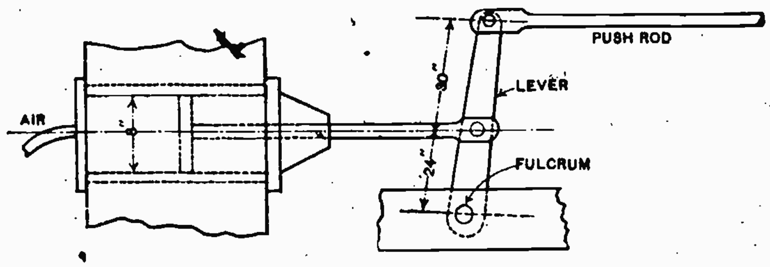


FIG. 31.

3. If the air pressure in the brake cylinder was 90 pounds per square inch, what would be the force transmitted by the push rod?

Fig. 32 shows an air hoist using a regular shop tackle with two pulleys in the lower, or movable, block and two in the upper block. The loose end of the rope is fastened to the upper block. This arrangement is useful when it is necessary to have a good deal of power with a low lift.

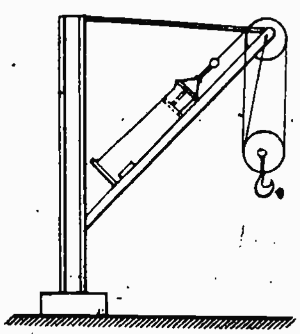


FIG. 32.

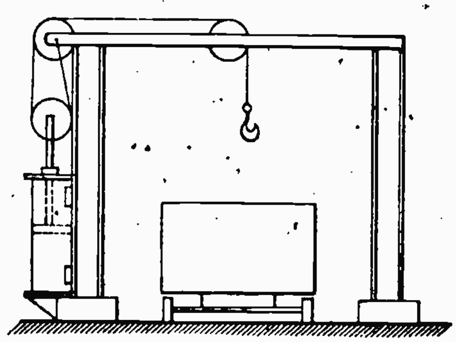


FIG. 33.

1. If the total lift required for the hoist shown in Fig. 33 is 8 feet and the heaviest casting weighs 12,000 pounds, what length of cylinder would we need, allowing 2 inches additional for clearance?

2. In the above problem, what power must we have on the piston?

3. The hoist shown in Fig. 33 has three pulleys—one on the piston rod and two on the structure. If it is required to lift a maximum load of 9,000 pounds with an available air pressure of 80 pounds per square inch, what size cylinder should be used? Give answer to nearest even inch.

SUBCOMMITTEE 3. PREPARATION OF TEACHERS OF MATHEMATICS FOR TRADE AND INDUSTRIAL SCHOOLS.

The committee charged with the investigation of the preparation of teachers consider the character of the preparation which teachers of mathematics have had who are now employed in trade schools or in schools which emphasize the practical element, and also the changes in preparation for this work which are desirable, and how such changes may be brought about. As a preliminary to the two main inquiries it was also thought to be desirable to investigate briefly the status of mathematics teaching in practical schools in order to determine the relative importance of the subject as indicated by the amount of time given to it and by the character of the teaching. With these objects in view about 25 schools were chosen, located in widely separated sections of the United States, including all the day and evening trade schools, public or private, known to the investigator, and also about the same number of technical or industrial schools having a more general vocational aim; and to these were sent the following questionnaire:

1. Is mathematics taught at all in your school? If so, kindly indicate the extent in hours per year (*a*) of theoretical mathematics; (*b*) of applied mathematics.

2. Is mathematics in your school taught by one or more specialists in that subject, by a teacher of other subjects, or by both classes of teachers? (*a*) Number of special teachers of mathematics; (*b*) number of teachers of other subjects who give mathematical instruction.

3. Outline briefly the character of the preparation which your teachers of mathematics have had, whether specialists or not.

4. What facilities for preparing teachers of mathematics for trade-school work are known to you?

5. Do you feel the necessity of extending the facilities for such preparation; and if so, what recommendations would you make in regard to it?

The responses to this questionnaire were prompt and generally complete. There was a wide range in the statistical replies called for in the first two questions, as might be expected from the varied character of the schools to which the questionnaire was sent. There would be no value in tabulating these returns. The significant facts determined were that nearly all the schools give some instruction in theoretical mathematics, only two reporting "no instruction," and all reported that they give some instruction in applied mathematics. The greatest number of hours of instruction in theoretical mathematics is 440 hours per year and in applied mathematics 988 hours per year. This is so far in excess of the average that it seemed somewhat abnormal. These figures were reported by the director of a

trade school connected with a certain large manufacturing interest. Aside from this extreme example the average time devoted to theoretical mathematics may be said to be about 300 hours covering a period of one and one-half years, while the average time devoted to applied mathematics is 210 hours covering about the same time. Several of the reports, however, recognize no distinction between the theoretical and the applied mathematics, but give the average amount of time to the subject as a whole. There is some reason in this view of the case, inasmuch as theoretical mathematics should be, and presumably is, taught in practical schools not for mental discipline, but for its utility as a necessary part of the equipment of the worker in a shop or factory. It is, in fact, very difficult to draw the line between theoretical mathematics taught with a practical aim and applied mathematics.

The replies to the second question indicated that about the same number of special teachers of mathematics are employed as those who teach mathematics as a part of their work. The reports indicate, however, a tendency in the direction of the employment of special teachers. In all large schools the special teacher is the rule, though one large industrial school reported only 3 special teachers of mathematics and 13 who teach other subjects as well. This is evidently an exceptional instance. Most of the schools reporting a mixed program for teachers of mathematics were small schools in which such a policy is a manifest necessity. There seemed to be recognition of the idea that mathematics teaching, to be most effective, should be in the hands of special teachers trained by study and by experience for this work.

Turning now to the first of the two main points of our inquiry viz, the character of the preparation which teachers of mathematics in practical schools have had, the reports indicate that college graduates and graduates of engineering schools largely predominate, though in a few instances the highest training was received in normal schools. It should be said, however, that all the reports emphasized the fact that, in the selection of teachers, much weight was given to practical experience in the shop or drafting room as a necessary element in the training of teachers of mathematics in practical schools. Such training is manifestly essential to effective work in applied mathematics. Speaking of the teachers of applied mathematics, the director of the School of Industrial Foremen in Boston says: "They are teachers of many years' experience in mechanical and electrical engineering subjects, but have not specialized in mathematical work. Such teachers are much better able to get the point of view of men carrying on work in the drafting room, shop, or factory than is possible for a teacher whose experience is limited to mathematical instruction pure and simple." This quotation fairly

represents the general tone of all the reports on this point. College graduates who are teachers of experience are effective in teaching theoretical mathematics; but in order to present the subject of applied mathematics effectively much actual shop experience is essential. This thought was well expressed in one of the papers by the director of the industrial classes in Fitchburg, Mass., in recommending preparation in the "best possible technical schools with 5 or 10 years' shop experience as apprentice or journeyman." But, ideal as such preparation may be, it will be recognized as a very difficult thing to accomplish in any large way. Certainly the number of teachers of mathematics, even in practical schools, who have received such training must be relatively small. With very few exceptions the answer to the fourth question—viz, What facilities for preparing teachers of mathematics for trade-school work are known to you?—was "None at all."

Passing to the second main point of inquiry—viz, What changes in the present order of preparation for the practical teaching of mathematics are desirable?—the trend of the replies was in favor of extending such scientific and practical training as engineering courses and shop experience can afford. No director of industrial or trade schools seems to feel that special school facilities for preparing teachers of mathematics for trade-school work would be desirable or indeed possible. One director writes that a course in practical mathematics with a view to teaching "should be given in engineering courses in technical colleges," and another says that it seems best that no special facilities to develop teachers of practical mathematics should be provided, but rather that the effort should be made to develop in engineering schools teachers who have a realizing sense of the importance of applied mathematics and who should further perfect themselves for teaching by actual experience in the solution of practical problems as they arise from time to time in actual shopwork.

Finally, this report may be summarized as follows: There are at present no special facilities for training teachers of mathematics for trade schools. Such teachers now find their best preparation, after the necessary theoretical groundwork, in several years of actual experience in applying mathematics to the problems of the shop and drafting room. This practical experience is considered to be of such importance that it does not seem desirable, from the present point of view of those who have had experience in directing trade-school work, to encourage special school training to develop teachers of mathematics for trade schools.

It is only fair to suggest that in some of our normal schools the kind of training which is given in mathematics will do much to equip teachers for work in industrial and trade schools. This is especially

true in schools in which much attention is given in the industrial arts. Not infrequently the teacher in training will have the opportunity to teach mathematics to students who are engaged in work in the household and industrial arts, and the demand will be made that problems growing out of their shopwork be used in the mathematics classes. Such training will not, of course, take the place of actual shop experience upon the part of the teacher, but we may expect that students who have had such experience will at least appreciate the need for knowledge of the work done in shops.

To the student of the elementary school curriculum these investigations concerning the type of work demanded in mathematics for special classes of children will be suggestive in the reorganization of the whole curriculum for special groups of children. It is undoubtedly true that in all subjects the tendency has been to provide a uniform course of study which takes little account of the lack of ability on the part of those who quit school and start to work because of the lack of interest in the school program. On the other hand, the school has made little or no provision for children of superior ability. If it is true that the curriculum all along the line needs to be differentiated in at least the last two years of the elementary school course to meet the needs of those who are to go into the industries, it is none the less true that special provision should be made for exceptionally capable children which will result in the saving of time in their preparation for their later work in college or technical schools. This investigation in the teaching of mathematics will, therefore, it seems, not only be significant in giving information to those who plan to organize work in this field, but it will also point the way for a reorganization which is demanded in other subjects.

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